Pre-Calculus Unit Plan:

Vectors and their Applications

Dr. Mohr-Schroeder

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University of Kentucky

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Andrea Meadors

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**Vectors in the Plane**

**Content:** This unit covers vectors in the plane, including component form, vector operations, unit vectors, direction angles, applications of vectors, the dot product, angles between two vectors, and finding vector components. These ideas are developed through exploration and discovery and traditional teaching methods combined with technology resources and hands-on activities that allow students to manipulate the vectors and see connections between the mathematics content and real-world situations. In this unit, these instructional strategies include patty paper activities that allow students to move the vectors to understand relationships between vectors of equal magnitude, direction, or both, and a GeoGebra activity that helps students to understand what a unit vector is and how it changes when the given vector changes. There is a discovery lesson over graphical representations of vector addition and multiplication, and a hands-on, multidisciplinary activity that allows students to see real-world statics (physics) applications to vectors. Throughout the unit, students are presented with applications to everyday life and many of the definitions and formulas are framed in a real-world setting.

**Sequence:** Prior to this unit, students will have studied the coordinate plane. They should understand not only how to graph points on a grid, but how to get the distance between the two points using the distance formula. Students will have learned about absolute value and be able to distinguish between the absolute value of a number and its actual signed value. Students should be comfortable with operations with integers. Students will also have learned trigonometric ratios and should know how to solve equations involving the ratios, especially sine, cosine, and tangent. This content is preparing students for multidisciplinary ideas involving vectors, particularly in physics, and is also useful in preparation for future mathematics courses many of these advanced students will take, especially Matrix Algebra and Calculus III.

**KCCSSM Practices:**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>MP. 1</th>
<th>MP. 2</th>
<th>MP. 3</th>
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<tr>
<td>Introduction to Vectors</td>
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<td>Operations on Vectors</td>
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<td>Direction Angles and Dot Product</td>
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<td>Vector Projections and Applications</td>
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<td>Vectors Exam</td>
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**Equity:** The lessons in this unit are prepared with a diverse student population in mind. English Language Learners benefit from guided notes, as well as opportunities to express mathematical ideas with correct mathematical language and grammatical structure. Students with specific learning needs are taken into consideration with mixed-ability groups. These groups provide students who need more time to go at their own pace. It also allows them the opportunity to ask questions in a smaller setting and to be paired with students who understand the material.
and can help explain it to them. Students who are traditionally strong also benefit from these groups because it gives them the opportunity to grow in their understanding of the material and to be able to explain the material to their peers. Students who are struggling mathematics learners will benefit from the step-by-step instructions in the exploration activities and the steps help them to build problem-solving and critical thinking skills.

**Learning Context and Implications:** This unit is designed for an eleventh grade advanced precalculus classroom in a suburban high school. It is important for students to be able to use their mathematical knowledge in real-world problem-solving and this unit provides students with a topic that is rich in multidisciplinary context. Students will use information about vectors in physics and will be able to use this knowledge in future mathematics courses. Students in Advanced Precalculus also need to be able to make connections between previously-learned mathematical content and new content. This unit requires students to use knowledge they have already acquired in order to solve new problems. The technology used in this unit help students to be able to make connections and to visualize abstract information. GeoGebraTube is a Geogebra-based website that has activities made using the free software and is used in the unit to help students to understand unit vectors. The SmartBoard will be used to present the dynamic GeoGebraTube activity, as well as provide a great resource for communicating with students and parents since the notes can be saved and uploaded to the class webpage for review. This is also great for special education students who need help with note-taking since this allows them to focus on the material in class and get the notes after class. The resources at an individual school may vary, however, and students may have more or less resources to work with. Certain graphing calculators can graph vectors, for example, and activities could be completed on the calculator. If there was no SmartBoard, the lesson could be presented in the traditional form, with copies of the notes given to students who need them. Some students may need longer review of the material, depending on the context of the classroom and the students’ funds of knowledge. This would make the unit longer with an additional day of review at the beginning for solving equations with trigonometric functions, graphing on a coordinate plane, etc. However, given the nature of the school setting, the advanced status of the course, and the students’ accelerated pace, it seems unlikely that accommodations that take away the technology or add time to the lesson would be necessary in the school for which this unit was designed.

**Key Concepts/Big Ideas/Essential Questions Focus:** The big ideas in the unit are number and quantity and modeling. The unit is designed to help students make connections to previously-understood number systems and their operations and properties (commutative, associative, and distributive properties) and to model real-world scenarios with mathematical language involving vectors. Students should be able to explain their model in terms of units and understand the relationship between the abstract ideas and the concrete information presented. The unit is designed with the Common Core State Standards for Mathematics in mind, and the alignment is listed below. This alignment shows that the unit is preparing the students with the material that Kentucky expects them to know and is doing so in an engaging manner that considers diverse student populations. The hands-on activities, technology
resources, and the group work involved in the unit are preparing students to be able to represent and model with vector quantities and perform operations on vectors.

**Statement of Objectives for the Unit and Alignment with Kentucky Core Academic Standards:**

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Objectives</th>
<th>CCSSM Alignment</th>
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| Component Form and Magnitude of Vectors | o  Students will distinguish and explain the difference between a vector and a scalar and can express the vector in component form, given an initial point and terminal point, and explain what the components describe.  
  o  Students will find the magnitude of a vector.  
  o  Students will find and determine equal vectors | Kentucky Core Academic Standards  
  N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.  
  N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. |
| Operations on Vectors               | o  Students will construct geometric representations of vector addition and multiplication of a vector by a scalar.  
  o  Students will identify patterns in the geometric representation of vector addition and multiplication of a vector by a scalar to construct rules about operations on vectors.  
  o  Students will use their knowledge about vector addition to prove that the magnitude of the sum of two vectors is not always equal to the sum of the magnitudes. | Kentucky Core Academic Standards  
  N-VM. 4. (+) Add and subtract vectors.  
  N-VM.5.(+) Multiply a vector by a scalar.  

## Mathematical Practices

- M.P.3 Construct viable arguments and critique the reasoning of others  
- M.P.6 Attend to precision.  
- M.P.7 Look for and make use of structure.
<table>
<thead>
<tr>
<th>Unit Vectors and Direction Angles</th>
<th>o Students will represent vector subtraction graphically by using the fact that ( v-w=v+(-w)=v+(-1)w ).</th>
<th>Kentucky Core Academic Standards</th>
<th>N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. N.VM.4 (+) Add and subtract vectors. N.VM.5 (+) Multiply a vector by a scalar.</th>
</tr>
</thead>
<tbody>
<tr>
<td>o Students will find a unit vector in the direction of the given vector.</td>
<td>Mathematical Practices</td>
<td>M.P.1 Make sense of problems and persevere in solving them. M.P.6 Attend to precision. M.P.7 Look for and make use of structure.</td>
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<tr>
<td>o Students will find a vector, ( v ), in the same direction as ( u ), given the magnitude.</td>
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<tr>
<td>o Students will find the magnitude and direction angle of a vector.</td>
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<tr>
<td>Direction Angles and Dot Product</td>
<td>o Students will find the component form of ( \vec{v} ) given its magnitude and the angle it makes with the positive x-axis.</td>
<td>Kentucky Core Academic Standards</td>
<td>N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. N.VM.5 (+) Multiply a vector by a scalar.</td>
</tr>
<tr>
<td>o Students will find the component form of the sum of ( \vec{u} ) and ( \vec{v} ) direction angles ( \theta_u ) and ( \theta_v ).</td>
<td>Mathematical Practices</td>
<td>M.P.1 Make sense of problems and persevere in solving them. M.P.6 Attend to precision. M.P.8 Look for and express regularity in repeated reasoning.</td>
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<tr>
<td>o Students will compute the dot product of two vectors and explore the properties of the dot product.</td>
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<tr>
<td>Vector Projections and Applications</td>
<td>o Students will describe the angle formed between two vectors given the two vectors and write an equation for ( \theta ) in</td>
<td>Kentucky Core Academic Standards</td>
<td>N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.</td>
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</tbody>
</table>
| terms of dot products and magnitudes  | N.VM.4 (+) Add and subtract vectors.  
| o Students will find and explain what is meant by the component of $\vec{u}$ along $\vec{v}$.  
| o Students will find the projection of $\vec{u}$ onto $\vec{v}$ and describe how it relates to the component of $\vec{u}$ along $\vec{v}$.  
| o Students will make real world connections to vectors, and use the properties of vectors to describe the forces acting on an object at rest.  
| N.VM.5 (+) Multiply a vector by a scalar.  

**Mathematical Practices**  
M.P.1 Make sense of problems and persevere in solving them.  
M.P.2 Reason abstractly and quantitatively.  
M.P.4 Model with mathematics  
M.P.5 Use appropriate tools strategically.  

| Vectors Exam  | Kentucky Core Academic Standards  
| o Students will find the component form and magnitude of the vector $\vec{v}$.  
| o Students will perform vector addition and scalar multiplication.  
| o Students will find unit vectors and direction angles of given vectors.  
| o Students will find the component form of $\vec{v}$ given its magnitude and the angle it makes with the positive x-axis.  
| o Students will find the dot product of two vectors and the angle between two vectors.  
| o Students will determine whether two vectors are orthogonal, parallel, or neither.  
| o Students will find the projection of $\vec{u}$ onto $\vec{v}$.  
| o Students will apply vectors to the real world.  
| N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.  
| N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.  
| N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.  
| N.VM.4 (+) Add and subtract vectors.  
| N.VM.5 (+) Multiply a vector by a scalar.  

**Mathematical Practices**  
M.P.1 Make sense of problems and persevere in solving them.  
M.P.2 Reason abstractly and quantitatively.  
M.P.6 Attend to precision.  
M.P.8 Look for and express regularity in repeated reasoning.
Communication with Students, Parents/Caregivers, Colleagues: Throughout the semester, students and parents can view the class website to gain access to notes, homework assignments, and make-up work necessary. Parents and students can view grades online on Infinite Campus for all assessments. Students will receive feedback throughout the unit on their assessments in the form of teacher comments and grades. Collaboration will take place among colleagues in the Professional Learning Community (PLC). Below is a summary of the communication before, during, and after instruction.

- **Before instruction:**
  - Students will receive an overview of the unit and will briefly be given real-world context for vectors so that they have an idea of what will be covered and how they will be assessed. It will be communicated to students that they can attend tutoring offered after school throughout the entire unit if they are struggling with the material.
  - Parents will have access to the website and any materials that students are given during the introduction to the unit.
  - Colleagues will be consulted to work out the timing details for the unit and to consult about assessments. Also, an expert in physics will also be consulted to discuss the unit and potential multidisciplinary links that might have previously been missed in creating the lessons.

- **During instruction:**
  - Students will receive feedback during in-class questioning and through grades and teacher comments on formative assessments given. Students can also access the course webpage at any time to get notes that were missed.
  - Parents can access the course webpage to view student assignments and assessments. They can also call or email to discuss grades or issues their child may be having in the class.
  - Colleagues will help discuss student progress throughout the unit on the formative assessments to make changes necessary to help reach the students.

- **After instruction:**
  - Students will receive feedback on their summative assessment and will receive supplementary instruction after the exams are graded on any exam questions that the class did poorly on as a whole. Students are welcome to discuss any problems they might have in an individual meeting with the teacher.
  - Parents of any student who receives below a C on the exam will be provided with a grade report that must be signed and returned to the class. Parents are always welcome to view their child’s grades online. Emails and calls are always welcome from parents who have a concern.
  - Colleagues will discuss the needs for changing the unit the next time it is taught, as well as student performance on the summative assessment.
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

### Component I: Classroom Teaching

#### Task A-2: Lesson Plan

**Intern Name:** Andrea Meadors, Stephen Powers, and Jessica Doering  
**Date:**  
**Cycle:**

**# of Students:**

**Age/Grade Level:** 11th and 12th  
**Content Area:** Pre-Calculus

**Unit Title:** Vectors and their Applications  
**Lesson Title:** Component Form and Magnitude of Vectors

#### Lesson Alignment to Unit

Respond to the following items:

a) Identify learning targets addressed by this lesson.

1. I can distinguish and explain the difference between a vector and a scalar and can express the vector in component form, given an initial point and terminal point, and explain what the components describe
2. I can find the magnitude of a vector.
3. I can find and determine equal vectors

b) Connect the learning targets to the state curriculum documents, i.e., Kentucky Core Academic Standards and the Mathematical Practices. List at least 2-3 target standards and at least 2 mathematical practices.

*Kentucky Core Academic Standards*

N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

*Mathematical Practices*

M.P.3 Construct viable arguments and critique the reasoning of others  
M.P.6 Attend to precision.  
M.P.7 Look for and make use of structure.

c) Describe students’ prior knowledge or focus of the previous learning.

The prior knowledge that this lesson will draw on are:
- Student understanding of the scalar quantities (length, mass, etc.).
- Students knowing the difference between distance and displacement.
- Student’s ability to plot points and use Pythagorean Theorem and the distance formula to find the length of a segment

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The summative assessment comes in the form of a test over vectors and their applications. This lesson provides a mathematical representation, using component form to describe vectors, which then makes it possible to talk about the applications

e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit.
Throughout the lesson, the teacher will have opportunities to give students individual help that need it. Those that require extra time to complete assignments as stated in their IEPs will receive it. The teacher will handle students with additional diverse needs on an individual basis.

f) Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

This lesson opens with a bell ringer that focuses on plotting points on the coordinate plane and then finding the distance between those points. The hope is that this will have these concepts fresh in their mind to correctly plot a vector and then find its magnitude.

<table>
<thead>
<tr>
<th>Lesson Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective/Target:</td>
<td>Assessment description: Student questioning and discovery based on prior knowledge. Exit Slip</td>
<td>Strategy/Activity: Lecture, both teacher and student led, and examples worked on the board.</td>
</tr>
<tr>
<td>I can distinguish and explain the difference between a vector and a scalar and can express the vector in component form, given an initial point and terminal point, and explain what the components describe.</td>
<td>Assessment Accommodations: Leading questions in the lesson to help promote student discussion.</td>
<td>Activity Adaptations: Guided notes will be available or provided</td>
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<td></td>
<td></td>
<td>Media/technologies/resources: N/A</td>
</tr>
<tr>
<td>Objective/Target:</td>
<td>Assessment description: Student questioning and student presented examples</td>
<td>Strategy/Activity: Lecture, both teacher and student led, and examples worked on the board.</td>
</tr>
<tr>
<td>I can find the magnitude of vectors.</td>
<td>Assessment Accommodations: The student discussion and presentation provides two separate methods for students to exhibit their knowledge.</td>
<td>Activity Adaptations: Guided notes will be available or provided</td>
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<td>Media/technologies/resources: Smart board</td>
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<tr>
<td>Objective/Target:</td>
<td>Assessment description: Worksheet to be filled out as student works through patty paper activity. Some questions require mathematical solutions, while others call for inferences made from results. (See attached activity documents)</td>
<td>Strategy/Activity: Worksheet to guide students through hands on representation of what creates equal vectors. Questions highlight connections between magnitude, direction, equal vectors, and component form.</td>
</tr>
<tr>
<td>I can find and determine equal vectors.</td>
<td>Assessment Accommodations: Activity is to be completed in partners in groups so worksheet can be completed together.</td>
<td>Activity Adaptations: Activity offers hands on manipulative to physically see equal vectors, but could also be done by using one of the dynamic geometry programs like Geogebra for students who need an alternative visual representation</td>
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<td>Media/technologies/resources: Patty Paper, Grid Paper, Computer and dynamic geometry software (if needed for accommodations)</td>
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Procedures: Describe the sequence of strategies and activities you will use to engage students and accomplish your objectives. Within this sequence, describe how the differentiated strategies will meet individual student needs and diverse learners in your plan. (Use this section to outline the who, what, when, and where of the instructional strategies and activities.)
I. Bell Ringer
   A. See attached bell ringer sheet
   B. Go over answers to bell ringer

II. Material Introduction
   A. What is a vector?
      a. What do we describe when we talk about things like length, mass, area, or temperature
         i) Ideal Answer: Some quantity, value, or how much
         ii) If Struggling: Have them provide examples of these and notice that they provide only a numerical value
   B. Distance vs. Displacement
      a. What is the difference?
         i) Ideal Answer: Distance is always positive.
         Follow up: Why is it that displacement can be negative and when does that happen?
         ii) If Struggling: Move to the number line description (examples below)
      b. Given that distance is a scalar and displacement is a vector, explain the distinction between a scalar and a vector.
         i) Ideal Answer: A vector includes a direction
         ii) If Struggling: Ask about the number line example. What does the negative displacement in part b refer to?

C. Geometric Description
   a. Definition: A vector, in the plane, is a line segment with an assigned direction. A "directed quantity"
      Examples: Draw line segments with arrows on board of various lengths and directions
   b. The examples show how to represent a vector, but we need a way to talk about them more precisely, so what would happen if we placed the line segment in the coordinate plane?

III. Component Form
   A. Notation
      a. Vector \( \vec{v} \)
      b. Initial point P- \((p_1, p_2)\)
      c. Terminal point Q- \((q_1, q_2)\)
         ORDER IS IMPORTANT!!!!!!!!!!!
   B. Finding the Components
      a. Component form describes how to get from Point P to Point Q.
         i) \( v_1 \) units over and \( v_2 \) units up
      b. In our drawing what does a represent?
         i) Change in x
         ii) \( v_1 = (q_1 - p_1) \)
      c. What about b?
         i) Change in y
         ii) \( v_2 = (q_2 - p_2) \)
      d. Component form- \( \vec{v} = (v_1, v_2) = ((q_1 - p_1), (q_2 - p_2)) \)
C. More notation
   a. This can also be expressed as $\vec{v} = v_1i + v_2j$
   b. Example- $\vec{v} = (2,4) = 2i + 4j$

D. Class Examples
   a. Done at the board, guided by students
      i) Find the component form of the vector $\vec{u}$ with initial point (-2,5) and terminal point (3,7)
         $$\vec{u} = \langle \Delta x, \Delta y \rangle$$
         $$= \langle (3-(-2), (7-5) \rangle$$
         $$= \langle 5,2 \rangle$$
   b. Done by students with a partner
      i) If $\vec{v} = (3,7)$ is drawn from initial point (2,4). What is the terminal point?
         $$\vec{v} = (3,7) = ((q_1-2), (q_2-4))$$
         $$q_1-2 = 3 \text{ and } q_2-4 = 7$$
         $$q_1 = 5 \text{ and } q_2 = 11$$
         So terminal point is (5,11)

IV. Magnitude
A. What is the magnitude of a vector?
   a. Initial Point $\rightarrow$ Terminal Point (Component Form) describes the direction of a vector.
   b. But a vector is a direction and a quantity. If component form describes the direction of a vector, what would describe the quantity (magnitude) of the vector?
      i) Ideal Answer- The length of the segment $\overline{PQ}$
      ii) If Struggling- Ask what is representing the direction, geometrically. Then ask how we would determine the magnitude of the line segment $\overline{PQ}$.
   c. The magnitude of a vector $\vec{v}$, denoted $|\vec{v}|$, is equal to length, or distance, from Point P to Point Q.
      $$||\vec{v}|| = |\overline{PQ}|$$

B. How can we find the magnitude of a vector $\vec{v}$?

Two Methods
a. Method 1- Distance formula.
   i) What information does the initial and terminal points provide.
      I) Ideal Answer- Point Q, $(p_1, p_2)$, and a Point Q, $(q_1, q_2)$, and we can use the distance formula,
         $$|\overline{PQ}| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$$
      II) If Struggling- Ask where are Point P and Point Q on the coordinate plane, and then ask how you would find the length of $\overline{PQ}$.
   ii) As shown in part I, show that the magnitude of $\vec{v}$ is equal to the distance from Point P to Point Q, distance formula.

b. Method 2- Pythagorean theorem.
   i) What information does component form provide? (refer to drawing on board)
      I) Ideal answer- The length of the change in x $(v_1)$ and the length of the change in y $(v_2)$.
      These are the two legs of a right triangle, so we know $v_1^2 + v_2^2 = ||\vec{v}||^2$ so $||\vec{v}|| = \sqrt{v_1^2 + v_2^2}$.
      II) If Struggling- ask them what a and b represent for the vector, and then have them look at the drawing and see that they are at a right angle.
   ii) As shown in part I, show that the magnitude of $\vec{v}$ is equal to the hypotenuse of a right triangle.

c. Make a Note
   i) Why do both of these methods work?
I) Component Form is given by \( \mathbf{v}_1 = q_1 - p_1 \) and \( \mathbf{v}_2 = q_2 - p_2 \)  

II) So by substitution \( \|\mathbf{v}\| = \sqrt{\mathbf{v}_1^2 + \mathbf{v}_2^2} = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = |\mathbf{PQ}| \) of more simply \( \|\mathbf{v}\| = |\mathbf{AB}| \)

C. Work Examples Individually (Find the magnitudes of the following vectors).

i) \( \mathbf{v} = \langle 2, -3 \rangle \) 
   \[ \|\mathbf{v}\| = \sqrt{2^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13} \]

ii) Vector with initial point, (1,0) and terminal point, (6,0) 
   \[ |\mathbf{PQ}| = \sqrt{(6 - 1)^2 + (0 - 0)^2} = \sqrt{5^2} = 5 \]

V. Magnitude Activity (Patty Paper)  
   A. See Attached Worksheet  
      a. To be completed in groups of two, three if necessary.

VI. If/Then Strategy  
   A. Have the students use extra sheets of patty paper to create vectors and then provide the initial and terminal points of equal vectors

VII. Closure  
   A. Complete Exit Slip  
      1. Answer Questions 3 and 4 from Activity sheet.
1. Give the coordinates for the following points.

Point A=(___ , ___)
Point B=(___ , ___)
Point C=(___ , ___)
Point D=(___ , ___)
Point E=(___ , ___)

2. Find the distances of the following segments
   a. \( \overline{AB} \)
   b. \( \overline{EC} \)
   c. \( \overline{BD} \)
Guided Notes
Vectors Day 1

I. Material Introduction
   A. What is a vector?
      a. What do we describe when we talk about things like length, mass, area, or temperature?
   B. Distance vs. Displacement
      a. What is the difference?
      b. Given that distance is a scalar and displacement is a vector, explain the distinction between a
         scalar and a vector.
   C. Geometric Description
      a. Definition- A vector, in the plane, is _________________________________. A
         “____________________”
         Examples:
      b. The examples show how to represent a vector, but we need a way to talk about them more
         precisely, so what would happen if we placed the line segment in the coordinate plane?

II. Component Form
   A. Notation
      a. Vector- _________
      b. Initial point P- _________
      c. Terminal point Q- _________

ORDER IS IMPORTANT!!!!!!!!!
B. Finding the Components
   
a. Component form describes how to get from Point P to Point Q.
   
b. In our drawing what does \( \mathbf{v}_1 \) represent?
   
   \[
   \mathbf{v}_1 = \underline{\phantom{0}}
   \]

c. What about \( \mathbf{v}_2 \)?
   
   \[
   \mathbf{v}_2 = \underline{\phantom{0}}
   \]

d. Component form- \( \mathbf{v} = \underline{\phantom{0}} \)

C. More notation
   
a. This can also be expressed as \( \mathbf{v} = \mathbf{v}_1 \mathbf{i} + \mathbf{v}_2 \mathbf{j} \)
   
   \[
   \mathbf{v} = \underline{\phantom{0}}
   \]

   b. Example- \( \mathbf{v} = (2, 4) = \underline{\phantom{0}} \)

D. Class Examples
   
a. Done at the board, guided by students
      i) Find the component form of the vector \( \mathbf{u} \) with initial point \((-2, 5)\) and terminal point \((3, 7)\)

b. Done by students with a partner
   i) If \( \mathbf{v} = (3, 7) \) is drawn from initial point \((2, 4)\). What is the terminal point?
III. Magnitude

A. What is the magnitude of a vector?
   a. Initial Point \( \rightarrow \) Terminal Point (Component Form) describes the direction of a vector.
   b. But a vector is a direction and a quantity. If component form describes the direction of a vector, what would describe the quantity (magnitude) of the vector?

   c. The magnitude of a vector \( \vec{v} \), denoted \( |\vec{v}| \), is equal to length, or distance, from Point P to Point Q.
      \[ ||\vec{v}|| = |\overrightarrow{PQ}| \]

B. How can we find the magnitude of a vector \( \vec{v} \)?
   Two Methods
   a. Method 1- Distance formula.
   b. Method 2- Pythagorean theorem.
      iii) What information does component form provide? (refer to drawing on board)

c. Make a Note
   ii) Why do both of these methods work?

C. Work Examples Individually (Find the magnitudes of the following vectors).
   i) \( \vec{v} = \langle 2, -3 \rangle \)
ii) Vector with initial point, (1,0) and terminal point, (6,0)
Patty Paper Activity
Finding Equal Vectors

*We know that vectors are equal if both their magnitude and direction are the same. For this activity we will use patty paper to determine if vectors are equal. On individual sheets of patty paper, use the graph paper to construct the following vectors on the patty paper (draw one vector per sheet of patty paper).*

1) Sketch and label the following vectors, along with the xy plane, given their initial and terminal points (remember one per sheet of patty paper).

\[ \vec{v} \rightarrow \text{Initial Point, (0,2) and Terminal Point, (1,4)} \]
\[ \vec{u} \rightarrow \text{Initial Point, (4,3) and Terminal Point, (6,4)} \]
\[ \vec{w} \rightarrow \text{Initial Point, (−1,3) and Terminal Point, (0,5)} \]
\[ \vec{q} \rightarrow \text{Initial Point, (0,7) and Terminal Point, (−2,6)} \]
\[ \vec{t} \rightarrow \text{Initial Point, (3,−3) and Terminal Point, (4,−1)} \]

2) Using your drawings, determine and record which vectors have equal magnitude.

3) Which of these vectors from #2 could be described as equal? Explain how why we know that these are equal.

4) Write each of the vectors in component form.

\[ \vec{v} = \underline{\quad} \]
\[ \vec{u} = \underline{\quad} \]
\[ \vec{w} = \underline{\quad} \]
\[ \vec{q} = \underline{\quad} \]
\[ \vec{t} = \underline{\quad} \]
5) Describe the relationship between the component form of the vectors that were found to be equal.

Critical Thinking Extension

a. What do you notice about the component form of the vectors $\vec{u}$ and $\vec{q}$? (*hint: how does this affect the magnitude?*)

b. Looking back at your patty paper constructions, what can be said about the direction of $\vec{u}$ and $\vec{q}$?

c. How would you mathematically describe the relationship between $\vec{u}$ and $\vec{q}$?
1. Answer to #3 from Activity

2. Answer to #4 from Activity
1. Give the coordinates for the following points.

Point A=(3,4)
Point B=(3,-2)
Point C=(-1,1)
Point D=(2,7)
Point E=(-5,-2)

2. Find the distances of the following segments
   a. $\overline{AB}$

   $|AB| = \sqrt{(3-3)^2 + (4-(-2))^2} = \sqrt{36} = 6$

   b. $\overline{EC}$

   $|EC| = \sqrt{(-5-(-1))^2 + (-2-1)^2} = \sqrt{25} = 5$

   c. $\overline{BD}$

   $|BD| = \sqrt{(2-3)^2 + (7-(-2))^2} = \sqrt{1^2 + 81^2} = \sqrt{82}$
Patty Paper Activity
Finding Equal Vectors

We know that vectors are equal if both their magnitude and direction are the same. For this activity we will use patty paper to determine if vectors are equal. On individual sheets of patty paper, use the graph paper to construct the following vectors on the patty paper (draw one vector per sheet of patty paper).

1) Sketch and label the following vectors, along with the xy plane, given their initial and terminal points (remember one per sheet of patty paper).

\[ \vec{v} \rightarrow \text{Initial Point, (0,2) and Terminal Point, (1,4)} \]
\[ \vec{u} \rightarrow \text{Initial Point, (4,3) and Terminal Point, (6,4)} \]
\[ \vec{w} \rightarrow \text{Initial Point, (−1,3) and Terminal Point, (0,5)} \]
\[ \vec{q} \rightarrow \text{Initial Point, (0,7) and Terminal Point, (−2,6)} \]
\[ \vec{t} \rightarrow \text{Initial Point, (3,−3) and Terminal Point, (4,−1)} \]

2) Using your drawings, determine and record which vectors have equal magnitude.

\[ |\vec{v}| = |\vec{u}| = |\vec{w}| = |\vec{q}| = |\vec{t}| \]

3) Which of these vectors from #2 could be described as equal? Explain how why we know that these are equal.

\[ \vec{v} = \vec{w} = \vec{t} \]

4) Write each of the vectors in component form.

\[ \vec{v} = (1,2) \]
\[ \vec{u} = (2,1) \]
\[ \vec{w} = (1,2) \]
\[ \vec{q} = (−2,−1) \]
\[ \vec{t} = (1,2) \]

5) Describe the relationship between the component form of the vectors that were found to be equal. They are identical

Critical Thinking Extension

a. What do you notice about the component form of the vectors \( \vec{u} \) and \( \vec{q} \)? (hint: how does this affect the magnitude?)

The components are negative, nothing.
b. Looking back at your patty paper constructions, what can be said about the direction of $\vec{u}$ and $\vec{q}$? They are in opposite directions of each other.

c. How would you mathematically describe the relationship between $\vec{u}$ and $\vec{q}$?

$$\vec{u} = -\vec{q}$$
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

**Component I: Classroom Teaching**

**Task A-2: Lesson Plan**

<table>
<thead>
<tr>
<th>Intern Name: Andrea Meadors, Stephen Powers, and Jessica Doering</th>
<th>Date:</th>
<th>Cycle:</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Students:</td>
<td>Age/Grade Level: 11th and 12th grade</td>
<td>Content Area: Pre-Calculus</td>
</tr>
<tr>
<td>Unit Title: Vectors and their Applications</td>
<td>Lesson Title: Vector Operations</td>
<td></td>
</tr>
</tbody>
</table>

**Lesson Alignment to Unit**

Respond to the following items:

a) Identify essential questions and/or unit objective(s) addressed by this lesson.

1. Students will construct geometric representations of vector addition and multiplication of a vector by a scalar.
2. Students will identify patterns in the geometric representation of vector addition and multiplication of a vector by a scalar to construct rules about operations on vectors.
3. Students will use their knowledge about vector addition to prove that the magnitude of the sum of two vectors is not always equal to the sum of the magnitudes.
4. Students will represent vector subtraction graphically by using the fact that \( v - w = v + (-w) = v + (-1)w \).

b) Connect the objectives to the state curriculum documents, i.e., Program of Studies, Kentucky Core Content, and/or Kentucky Core Academic Standards.

N-VM. 4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction \( v - w \) as \( v + (-w) \), where \(-w\) is the additive inverse of \( w\), with the same magnitude as \( w\) and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5.(+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as \( c(vx, vy) = (cvx, cvy) \).

**Standards for Mathematical Practice:**

1. (1) Make sense of problems and persevere in solving them.
2. (3) Construct viable arguments and critique the reasoning of others.
3. (7) Look for and make use of structure.

c) Describe students’ prior knowledge or focus of the previous learning.

Students know how to graph points on the coordinate plane. The previous lesson covered graphing vectors and writing them in component form. Supposedly, students know how to add and multiply.

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The summative assessment will include problems that will ask students to compute vector addition and multiply a vector by a scalar geometrically and in component form. Students will practice the computations and will derive formulas for general rules. The formative assessments in this lesson (activity, exit slip, and homework) will provide data for the students and
e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit.

Students in my classroom have diverse learning styles and a few have special learning needs, including English Language Learners. The students who require special modifications will be assisted per the student-teacher contracts or Individual Education Plans of the individual student. As usual, differentiated instruction for this lesson will take into consideration all of the students in the class, including the strongest. Mainly, students’ individual needs will be met by placing them in groups that are pre-determined by the pre-assessment data. Smaller groups will give students who are less likely to speak in the larger group an opportunity to participate and ask questions. The groups can move at their own pace. Also, the strongest students in the class will have the opportunity to teach their peers—a learning strategy that is used.

f) Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

Students will be given an exit slip the class before to measure understanding of the previous material. Gaps will be identified and discussed during the warm-up at the beginning of class.

<table>
<thead>
<tr>
<th>Lesson Objectives/ Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective/target:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Students will construct geometric representations of vector addition and multiplication of a vector by a scalar.</td>
<td>Assessment description: Students will be assessed through a warm-up activity, student questioning, group work, presentation of group work, and homework. Assessment Accommodations: Mixed-ability grouping will allow students who are struggling to get more individual attention from peers who have a stronger understanding. Students who have a better understanding of the material will be given an opportunity to further understand the material when they are asked to explain it to their peers. This will also give the teacher an indication of who has an understanding and who does not when walking around the room and listening to the conversations. The homework will include accommodations as per individual student contracts or IEP’s.</td>
<td>Strategy/Activity: Warm-up: Students will be asked to write vectors in component form given initial and terminal points and will be asked to graph vectors given in component form. Group work: Students will use the pair of vectors in the warm-up to explore how to graph addition of vectors geometrically. Presentation of group work: Students will be asked to present the work from their groups on their discoveries from the group work. Activity Adaptations: Mixed-ability grouping. Media/technologies/resources: SmartBoard, worksheet</td>
</tr>
<tr>
<td>Objective/target:</td>
<td>Assessment description:</td>
<td>Strategy/Activity:</td>
</tr>
<tr>
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</tr>
<tr>
<td>2. Students will identify patterns in the geometric representation of vector addition and multiplication of a vector by a scalar to construct rules about operations on vectors.</td>
<td>Students will be assessed student questioning, group work, presentation of group work, and homework.</td>
<td>Group work: Students will be asked to use the fact that multiplication is repeated addition to understand how to graphically represent multiplying a vector by a scalar. Presentations: Students will present their findings and will discuss the solutions presented.</td>
</tr>
<tr>
<td>Assessment Accommodations:</td>
<td>Same accommodations as objective #1.</td>
<td></td>
</tr>
<tr>
<td>Objective/target:</td>
<td>Assessment description:</td>
<td>Strategy/Activity:</td>
</tr>
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<tr>
<td>3. Students will use their knowledge about vector addition to prove that the magnitude of the sum of two vectors is not always equal to the sum of the magnitudes.</td>
<td>Homework</td>
<td>Students will be asked to use critical thinking and apply knowledge from the lesson to find an example of two vectors for which the statement is not true. Activity Adaptations:</td>
</tr>
<tr>
<td>Assessment Accommodations:</td>
<td>Students with special needs will be given homework sheets that will have more step-by-step help on the problems which require more of a “proofs” approach. Other individual accommodations will take place at the individual student level.</td>
<td>For all students, the homework sheet will be discussed in-class and (if time) they can begin working on their assignment.</td>
</tr>
<tr>
<td>Objective/target:</td>
<td>Assessment description:</td>
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<tr>
<td>-------------------</td>
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<tr>
<td>4. Students will represent vector subtraction graphically by using the fact that ( v-w = v + (-w) = v + (-1)w ).</td>
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<td>Students will be asked to use critical thinking and apply knowledge from the lesson to find an example of two vectors for which the statement is not true. Activity Adaptations:</td>
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**Procedures:** Describe the sequence of strategies and activities you will use to engage students and accomplish your objectives. Within this sequence, describe how the differentiated strategies will meet individual student needs and diverse learners in your plan. (Use this section to outline the who, what, when, and where of the instructional strategies and activities.)

**Opener/Warm-up Activity: (5 minutes)**

Students will be given a worksheet with two problems when they enter class. The first will ask them to graph two vectors that are given in component form. The second will ask them to write the component form of two vectors given the initial and terminal points and then graph the vectors.

Teacher will walk around the room while students are completing their worksheet to be sure everyone is attempting the problem and to assist struggling students.

Once about three minutes is up, students will get into groups (teacher-assigned mixed-ability groups based on pre-assessment data). They will compare their work to group members to correct errors.

**Lesson Development: (40 minutes)**

- **Group work: (5 minutes)**

  Before students begin group work, remind them of the classroom rules regarding working together in groups. This includes everyone working together in the group, not jumping ahead and not leaving anyone behind.

  Students will use the warm-up problem #1 to create a real-world problem that could be represented by the vectors. *Suggestions for students: airplane vs. the wind, two people tugging on a rope with different force in different directions.* They will then be asked to use that real-world problem to come up with how they would represent the addition of the two vectors geometrically. As groups finish, they will go to the board and write their group’s ideas.

- **Presentations of group work: (10 minutes)**

  Students will present their group’s ideas and once all groups have presented, the class will discuss the solutions in context of real-world problems. The class will then pick representations that make sense. Students will add the notes on the two proper ways to add vectors to their worksheets once the valid solutions are identified. The tail-to-tail method and the triangle method will be discussed.

- **Group work: (20 minutes)**

  After being reminded that multiplication is repeated addition, students will be asked to graph a vector, $\mathbf{v}$, on their worksheet. Then they will determine how to geometrically represent multiplying a vector by a scalar. They will be asked to graph $2\mathbf{v}$, $3\mathbf{v}$, and $4\mathbf{v}$.

  Walking around, the teacher can correct student errors and assist students by asking them questions to lead them to think critically and answer the question on their own. *For example, if a student did not understand the hint that multiplication is...*
repeated addition, they could be asked to write out what $2x$ means (namely, $x+x$). Then, they would be asked what $2v$ would mean, then, in light of what they just showed. Hopefully, they will see that they can just “add” the vector to itself using the addition methods found in the first part of the activity.

They will then be asked to find the component form of the resulting vectors from their addition and multiplication problems. Their worksheet will ask them if they notice any pattern when they compare the component forms of the original vectors with the component form of the resulting vectors.

- Presentation of group work: (5 minutes)

Class discussion will take place about the answers to the previous questions about the patterns in the component form. Ideas will be written on the board by the teacher. The correct method of adding two vectors in component form will be identified through class discussion and the notes will be added to the worksheet. Similarly, the correct method of multiplying a vector by a scalar in component form will be identified and added to the worksheet.

**Closure: (10 minutes)**

Students will be introduced to the homework questions that are given on the worksheet and the homework for next class will be assigned (p. 520 #1-21 odd). They will be given an exit slip that requires them to use the new rules given to add two vectors in component form and to multiply a vector in component form by a scalar. Additionally, the exit slip will include a space for them to describe, in words, how to add vectors using one of the two methods. This literacy strategy will not only check for comprehension, but will give students the opportunity to express the concepts they learned in class in mathematically correct language.

**If-Time Strategy:**

Students will be allowed to begin the discovery part of their homework sheet if there is additional time at the end of class.
Vector Operations

1. Write the following vectors in component form:
   \( \mathbf{u} \): Initial point \((3, -2)\), Terminal point: \((3, 3)\)
   \( \mathbf{u} = (\underline{\quad}, \underline{\quad}) \)

   \( \mathbf{v} \): Initial point \((2, 2)\), Terminal point: \((-1, 4)\)
   \( \mathbf{v} = (\underline{\quad}, \underline{\quad}) \)

   Now, graph \( \mathbf{u} \) and \( \mathbf{v} \):

2. Graph the following vectors below:
   \( \mathbf{u} = (2, 1) \)
   \( \mathbf{v} = (1, 3) \)

3. Create a real-world problem that is represented by the vectors in Graph #1. (Hint: Think about an airplane that is traveling in one direction, but the wind is pushing another direction. Or, think about two people pulling different directions on a rope.) Describe the real-world situation here:

   What does \( \mathbf{u} \) represent?

   What does \( \mathbf{v} \) represent?

4. Now, use that real-world situation that you described above to think about what you think that the graph of the resulting vector would look like if you added the two vectors together. (For example, if you did an airplane and wind, what direction do you think that the airplane would actually travel if the wind was blowing it off course?) Graph some ideas here:
Notes on Representing Vector Addition Graphically:

Method 1: ______________________

Method 2: ______________________

1. Remembering that multiplication is repeated addition, graph the vectors \( \mathbf{v} = (2,1) \), \( 2\mathbf{v} \), \( 3\mathbf{v} \), and \( 4\mathbf{v} \).
1. Write the component form of the vectors from the following problems:

   #1: \( u = (__, __), \ v = (__, __), \ u + v = (__, __) \)
   #2: \( u = (__, __), \ v = (__, __), \ u + v = (__, __) \)

   #5: #1: \( v = (1,2), \ 2v = (__, __), \ 3v = (__, __), \ 4v = (__, __) \)

2. Looking at the vectors above, do you notice any patterns when we look at the component form of the given vectors against the component form of the resulting vectors \( \vec{u} + \vec{v} \), \( 2\vec{v}, 3\vec{v}, 4\vec{v} \)?

   Can you make a conjecture about what you think could be a general formula for adding two vectors and multiplying a vector by a scalar?

---

**Notes:**

Given two vectors, \( \vec{u} \) and \( \vec{v} \), if \( \vec{u} = (u_1, u_2), \ \vec{v} = (v_1, v_2) \),

\( \vec{u} + \vec{v} = (__, __) \)

Given a vector, \( \vec{v} = (v_1, v_2) \) and a scalar \( k \) :

\( k\vec{v} = (__, __) \)
Vector Operations Exit Slip

1. Using either the triangle method or the parallelogram law, graph the vector $\mathbf{u} + \mathbf{v}$:
   
   $\mathbf{u} = (-5, 3), \mathbf{v} = (0, 0)$.

2. Let $\mathbf{v} = (2, 1)$. Graph the vectors $2\mathbf{v}$ and $5\mathbf{v}$.

3. Describe, in your own words, the parallelogram law and the triangle rule for adding vectors.
6.3 Exercises

In Exercises 1–12, find the component form and the magnitude of the vector \( v \).

1. \( y \)
   \[
   \begin{array}{c}
   (3, 2) \\
   \end{array}
   \]

2. \( y \)
   \[
   \begin{array}{c}
   (-4, -2) \\
   \end{array}
   \]

3. \( y \)
   \[
   \begin{array}{c}
   (-1, 4) \\
   \end{array}
   \]

4. \( y \)
   \[
   \begin{array}{c}
   (3, 5) \\
   \end{array}
   \]

5. \( y \)
   \[
   \begin{array}{c}
   (3, 3) \\
   \end{array}
   \]

6. \( y \)
   \[
   \begin{array}{c}
   (-4, -1) \\
   \end{array}
   \]

Initial Point | Terminal Point
---|---
7. \((-1, 5)\) | \((15, 12)\)
8. \((1, 11)\) | \((9, 3)\)
9. \((-3, -5)\) | \((5, 1)\)
10. \((-3, 11)\) | \((9, 40)\)
11. \((1, 3)\) | \((-8, -9)\)
12. \((-2, 7)\) | \((5, -17)\)

In Exercises 13–18, use the figure to sketch a graph of the specified vector.

13. \(-v\) 14. \(5v\)
15. \(u + v\) 16. \(u - v\)
17. \(u + 2v\) 18. \(v - \frac{1}{2}u\)

In Exercises 19–26, find (a) \(u + v\), (b) \(u - v\), and (c) \(2u - 3v\). Then sketch your resultant vector.

19. \(u = \langle 2, 1\rangle\), \(v = \langle 1, 3\rangle\)
20. \(u = \langle 2, 3\rangle\), \(v = \langle 4, 0\rangle\)
21. \(u = \langle -5, 3\rangle\), \(v = \langle 0, 0\rangle\)
22. \(u = \langle 0, 0\rangle\), \(v = \langle 2, 1\rangle\)
23. \(u = \langle 1 + j\rangle\), \(v = \langle 2i - 3j\rangle\)
24. \(u = \langle -2i + j\rangle\), \(v = \langle -1 + 2j\rangle\)
25. \(u = \langle 2i\rangle\), \(v = \langle j\rangle\)
26. \(u = \langle 3j\rangle\), \(v = \langle 2i\rangle\)

In Exercises 27–36, find a unit vector in the direction of the given vector.

27. \(u = \langle 3, 0\rangle\) 28. \(u = \langle 0, -2\rangle\)
29. \(v = \langle -2, 2\rangle\) 30. \(v = \langle 5, -12\rangle\)
31. \(v = \langle 4i - 2j\rangle\) 32. \(v = \langle 1 + j\rangle\)
33. \(w = \langle 4j\rangle\) 34. \(w = \langle -6i\rangle\)
35. \(w = \langle 1 - 2j\rangle\) 36. \(w = \langle 7j - 3i\rangle\)

In Exercises 37–40, find the vector \(v\) with the given magnitude and the same direction as \(u\).

|\(v\)| |\(u\)|
---|---|
37. \|\(v\| = 5 | \(u = \langle 3, 3\rangle\)
38. \|\(v\| = 6 | \(u = \langle -3, 3\rangle\)
39. \|\(v\| = 9 | \(u = \langle 2, 5\rangle\)
40. \|\(v\| = 10 | \(u = \langle -10, 0\rangle\)

In Exercises 41–46, find the component form of \(v\) and sketch the specified vector operations geometrically, where \(u = 2i - j\) and \(w = i + 2j\).

41. \(v = \frac{2}{3}u\) 42. \(v = \frac{1}{3}w\)
43. \(v = u + 2w\) 44. \(v = -u + w\)
45. \(v = \frac{1}{2}(3u + w)\) 46. \(v = u - 2w\)

In Exercises 47–50, find the magnitude and direction angle of the vector \(v\).

47. \(v = 3(cos 60^\circ + sin 60^\circ)\)
47. 46,837.5 square feet
49. False. For \( s \) to be the average of the lengths of the three sides of the triangle, \( s \) would be equal to \((a + b + c)/3\).
51. Answers will vary.
53. 405.2 feet
55. Answers will vary. 
57. \( \frac{\pi}{2} \) \n59. \( \frac{\pi}{3} \) 
61. \( \frac{5\pi}{6} \)
63. \( \frac{\sqrt{1 - 9x^2}}{3x} \) 
65. \( \frac{\sqrt{4 - (x - 1)^2}}{2} \)
67. \( \sin \theta = -\frac{\sqrt{2}}{2} \) 
69. 12 = 6 sec \( \theta \)
71. \( \cos \theta = \frac{1}{2} \) 
\( \sec \theta = \frac{2}{\sqrt{3}} \) 
\( \csc \theta = -\frac{2}{\sqrt{3}} \) 
\( \csc \theta = \frac{2}{\sqrt{3}} \)

Section 6.3 (page 520)
1. \( v = (3, 2); [v] = \sqrt{13} \) 
3. \( v = (-3, 2); [v] = \sqrt{13} \) 
5. \( v = (0, 5); [v] = 5 \) 
7. \( v = (16, 7); [v] = \sqrt{505} \) 
9. \( v = (8, 6); [v] = 10 \) 
11. \( v = (-9, -12); [v] = 15 \) 
13. 
15.
17. 
23. (a) \( 3i - 2j \) 
(b) \(-1 + 4j \)
1. Write the following vectors in component form:
   \( \mathbf{u} \): Initial point \((3, -2)\), Terminal point: \((3, 3)\)
   \[ \mathbf{u} = (0, 5) \]
   \( \mathbf{v} \): Initial point \((2, 2)\), Terminal point: \((-1, 4)\)
   \[ \mathbf{v} = (-3, 4) \]
   Now, graph \( \mathbf{u} \) and \( \mathbf{v} \):

2. Graph the following vectors below:
   \[ \mathbf{u} = (2, 1) \]
   \[ \mathbf{v} = (1, 3) \]

3. Create a real-world problem that is represented by the vectors in Graph #1. (Hint: Think about an airplane that is traveling in one direction, but the wind is pushing another direction. Or, think about two people pulling different directions on a rope.) Describe the real-world situation here:
   What does \( \mathbf{u} \) represent? \textit{Answers will vary.}
   What does \( \mathbf{v} \) represent?

4. Now, use that real-world situation that you described above to think about what you think that the graph of the resulting vector would look like if you added the two vectors together. (For example, if you did an airplane and wind, what direction do you think that the airplane would actually travel if the wind was blowing it off course?) Graph some ideas here: \textit{Answers will vary.}
Notes on Representing Vector Addition Graphically:

Method 1: **Parallelogram Law/Tail-to-Tail Method**

Method 2: **Triangle /Tip-to-Tail Method**

(Parallelogram Law): "If two vector quantities are represented by two adjacent sides or a parallelogram then the diagonal of parallelogram will be equal to the resultant of these two vectors."

(Triangle Method): “If the initial point of the second vector is placed on the terminal point of the first, the resulting vector is given by a vector drawn from the initial point of the first vector and the terminal point of the second.”

1. Remembering multiplication is addition, graph the \( v = (2,1), 2v, \) and \( 3v. \)
1. Write the component form of the vectors from the following problems:

   \#1: \mathbf{u} = \langle 2,1 \rangle , \mathbf{v} = \langle 1,3 \rangle , \mathbf{u} + \mathbf{v} = \langle 3,4 \rangle \\
   \#2: \mathbf{u} = \langle 0,5 \rangle , \mathbf{v} = \langle -3,2 \rangle , \mathbf{u} + \mathbf{v} = \langle -3,7 \rangle \\
   \#5: \#1: \mathbf{v} = \langle 2,1 \rangle , 2\mathbf{v} = \langle 4,2 \rangle , 3\mathbf{v} = \langle 6,3 \rangle \\

2. Looking at the vectors above, do you notice any patterns when we look at the component form of the given vectors against the component form of the resulting vectors (\mathbf{u} + \mathbf{v}, 2\mathbf{v}, 3\mathbf{v}, 4\mathbf{v})? 
   (Answers may vary.)

   Can you make a conjecture about what you think could be a general formula for adding two vectors and multiplying a vector by a scalar? 
   (Answers may vary.)

**Notes:**
Given two vectors, \( \mathbf{u} \) and \( \mathbf{v} \), if \( \mathbf{u} = \langle u_1, u_2 \rangle \), \( \mathbf{v} = \langle v_1, v_2 \rangle \),

\[
\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle
\]

Given a vector, \( \mathbf{v} = \langle v_1, v_2 \rangle \) and a scalar \( k \) :

\[
k\mathbf{v} = \langle kv_1, kv_2 \rangle
\]
1. Using either the triangle method or the parallelogram law, graph the vector $u + v$:
   $u = (-5,3), v = (1,1)$.

\[ \text{Diagram of vector addition} \]

2. Let $v = (2,1)$. Graph the vectors $2v$ and $5v$.

\[ \text{Diagram of scaled vectors} \]

(Scale on x-axis: 2:1).

3. Describe, in your own words, the parallelogram law and the triangle rule for adding vectors.

The parallelogram law: "If two vector quantities are represented by two adjacent sides or a parallelogram then the diagonal of parallelogram will be equal to the resultant of these two vectors."
The triangle method: “If the initial point of the second vector is placed on the terminal point of the first, the resulting vector is given by a vector drawn from the initial point of the first vector and the terminal point of the second.”
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

**Component I: Classroom Teaching**

**Task A-2: Lesson Plan**

<table>
<thead>
<tr>
<th>Intern Name:</th>
<th>Andrea Meadors, Stephen Powers, and Jessica Doering</th>
<th>Date:</th>
<th>Cycle:</th>
</tr>
</thead>
</table>

**# of Students:**

<table>
<thead>
<tr>
<th>Age/Grade Level:</th>
<th>Content Area:</th>
</tr>
</thead>
<tbody>
<tr>
<td>11th and 12th</td>
<td>Pre-Calculus</td>
</tr>
</tbody>
</table>

**Unit Title:** Vectors and their Applications  
**Lesson Title:** Unit Vectors and Direction Angles

### Lesson Alignment to Unit

Respond to the following items:

a) Identify learning targets addressed by this lesson.

1. Students will find a unit vector in the direction of the given vector.
2. Students will find a vector, \( \vec{v} \), in the same direction as \( \vec{u} \), given the magnitude.
3. Students will find the magnitude and direction angle of a vector.

b) Connect the learning targets to the state curriculum documents, i.e., Kentucky Core Academic Standards and the Mathematical Practices. List at least 2-3 target standards and at least 2 mathematical practices.

**Kentucky Core Academic Standards**

- N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
- N.VM.4 (+) Add and subtract vectors.
- N.VM.5 (+) Multiply a vector by a scalar.

**Mathematical Practices**

- M.P.1 Make sense of problems and persevere in solving them.
- M.P.6 Attend to precision.
- M.P.7 Look for and make use of structure.

c) Describe students’ prior knowledge or focus of the previous learning.

Students have knowledge of vectors including how to find component form of a vector, how to find magnitude of a vector, how to graph vectors, how to perform vector addition, and how to multiply a vector by a scalar.

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The summative assessment comes in the form of a test over vectors and their applications. This lesson provides a mathematical representation, using unit vectors and direction angles, which then make it possible to talk about the applications of vectors.

e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit.

Throughout the lesson, the teacher will have opportunities to give students individual help that need it. Those that require extra time to complete assignments as stated in their IEPs will receive it. The teacher will handle students with additional diverse needs on an individual basis.
f) Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

The warm-up will provide a brief review of the previous day’s lesson. The lesson will build upon previously learned concepts such as how to find the magnitude of a vector and how to multiply a vector by a scalar.

<table>
<thead>
<tr>
<th>Lesson Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
</thead>
</table>
| **Objective/target:** 1. Students will find a unit vector in the direction of the given vector. | **Assessment description:** Student questioning, board work, homework.  
**Assessment Accommodations:** Built in questions that students may have, and individual work gives time to talk with students. | **Strategy/Activity:** Guided notes and individual work  
**Activity Adaptations:** While students work individually, the teacher can focus on students who need extra help.  
**Media/technologies/resources:** Document Camera, Smartboard, GeogebraTube and Worksheet |
| **Objective/target:** 2. Students will find a vector, \( \mathbf{v} \), in the same direction as \( \mathbf{u} \), given the magnitude. | **Assessment description:** Student questioning, board work, homework, exit slip.  
**Assessment Accommodations:** Built in questions that students may have, and individual work gives time to talk with students. Some students may require extra time for their exit slip. | **Strategy/Activity:** Guided notes and individual work  
**Activity Adaptations:** While students work individually, the teacher can focus on students who need extra help.  
**Media/technologies/resources:** Document Camera and Worksheet |
| **Objective/target:** 3. Students will find the magnitude and direction angle of a vector. | **Assessment description:** Student questioning, board work, homework, exit slip.  
**Assessment Accommodations:** Built in questions that students may have, and individual work gives time to talk with students. Some students may require extra time for their exit slip. | **Strategy/Activity:** Guided notes and individual work  
**Activity Adaptations:** While students work individually, the teacher can focus on students who need extra help.  
**Media/technologies/resources:** Document Camera and Worksheet |

Procedures: Describe the sequence of strategies and activities you will use to engage students and accomplish your objectives. Within this sequence, describe how the differentiated strategies will meet individual student needs and diverse learners in your plan. (Use this section to outline the who, what, when, and where of the instructional strategies and activities.)

**Opener/Warm-up Activity (5 minutes):**

Hand out warm-up worksheet (see attached) with one multi-step problem as students enter the classroom. The problem is a brief review of yesterday’s lesson: vector addition, scalar multiplication, and graphing vectors.

Walk around the room and stamp students’ papers as they are finished or nearly finished with their work. This is to provide a quick reference for assigning grades.
Reveal the correct work and answers on the SmartBoard for students’ to check their work. The students will place their stamped and corrected (if needed) papers into their binders to be checked at the end of the unit.

**Lesson Development (40 minutes):**

Hand out guided notes (see attached). Display notes on a document camera to be filled in along with the students throughout the lesson.

“Think about it” – Students will talk with a partner about possible answers to the “think about it” questions in the guided notes. Ask students what they think the correct answers are. Whether correct or incorrect, display the following virtual manipulative onto the Smartboard: [http://www.geogebratube.org/material/show/id/3550](http://www.geogebratube.org/material/show/id/3550) and use it to visually demonstrate what a unit vector is. Ask a student to come to the board and move the vector around to see that the unit vector never changes magnitude.

**Unit Vectors** - Read the definition of a unit vector from the notes and explain that a unit vector is the only vector in a given direction with a magnitude of 1, yet there are infinitely many vectors in a given direction of other magnitudes.

Find a unit vector in the direction of the given vector- After reading the formula for finding a unit vector, work Ex1 as a class, asking for participation along the way. Students will work Ex2 on their own. Circulate to check their progress and help those who need assistance. Call on a student whose work is correct to write the problem on the board and explain their process. Correct them when needed.

Work Ex3 as a class, asking for participation along the way. Students will work Ex4 on their own. Circulate to check their progress and help those who need assistance. Call on a student whose work is correct to write the problem on the board and explain their process. Correct them when needed.

**Linear Combinations** - Read definition of standard unit vectors and draw i and j on the board to demonstrate. Read formula for linear combinations. Work Ex5 as a class, asking for participation along the way. Work Ex6 as a class, asking for participation along the way.

Direction Angles - Read notes and notation for direction angles. Prompt students to help fill in missing information rather than just giving it to them. Refer to diagram in notes for a visual demonstration of the concept. Students will work Ex7 (parts a and b) on their own. Circulate to check their progress and help those who need assistance. Call on two different students who have correct work, but haven’t been as vocal in class to write each problem on the board and explain their process. Correct them when needed.

**Closure (10 minutes):**

Hand out exit slip (see attached) that requires them to demonstrate their understanding of unit vectors and direction angles. They will turn it in to the appropriate tray for their class and copy down assigned homework problem numbers from their textbook (see attached).

**If-Time Strategy:**

Students will begin reading the next section of their textbook and hypothesize uses for direction angles in anticipation of the upcoming lesson focusing on vector applications.
Using the vectors below, find (a) \( \mathbf{u} + \mathbf{v} \), (b) \( \mathbf{u} - \mathbf{v} \), and (c) \( 3\mathbf{u} - 2\mathbf{v} \). Then sketch your resultant vector.

\[
\mathbf{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}
\]
Formulas learned so far

Component Form of a Vector: \[ \mathbf{v} = \mathbf{PQ} = (q_1 - p_1, q_2 - p_2) = (v_1, v_2) \]

Magnitude of a vector: \[ \|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2} \]

Think about it

How many possible vectors can you find in one given direction? _________________________

How many possible vectors can you find in one given direction that have a magnitude of 1? _________________________

A unit vector is a vector that has the same direction of a vector \( \mathbf{v} \) and a magnitude of 1.

Note: A unit vector is usually indicated with a \( \hat{\ } \) over the letter denoting the vector, for example: \( \hat{\mathbf{u}} \)

It is often useful to find a unit vector in the same direction as a given vector \( \mathbf{v} \). To do this \( \hat{\mathbf{v}} \) by its magnitude.

\[ \hat{\mathbf{u}} = \text{unit vector} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left( \frac{1}{\|\mathbf{v}\|} \right) \mathbf{v} \]

Ex1) Find a unit vector in the direction of \( \mathbf{v} = (-3, 5) \) and verify that the resulting vector has a magnitude of 1.

Ex2) Find a unit vector in the direction of \( \mathbf{v} = (5, -12) \) and verify that the resulting vector has a magnitude of 1.
Ex3) Find the vector $\mathbf{v}$ with the given magnitude and the same direction as $\mathbf{u}$ and sketch the vector.

$\|\mathbf{v}\| = 5 \quad \mathbf{u} = \langle 3, 3 \rangle$

Ex4) Find the vector $\mathbf{v}$ with the given magnitude and the same direction as $\mathbf{u}$ and sketch the vector.

$\|\mathbf{v}\| = 9 \quad \mathbf{u} = \langle 2, 5 \rangle$

The unit vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$ are called the **standard unit vectors** and are denoted by $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

Any vector in the plane can be written as a linear combination of the standard unit vectors.

**Linear Combination:**

$\mathbf{v} = v_1 \mathbf{i} + v_2 \mathbf{j}$

Ex5) Write the vector $\mathbf{v} = \langle -5, 3 \rangle$ as a linear combination.
Ex6) Let \( u = -3i+8j \) and \( v = 2i-j \). Find \( 2u-3v \).

If \( \hat{u} \) is a unit vector such that \( \theta \) is the angle (measured clockwise) from the positive x-axis to \( \hat{u} \), the terminal point of \( \hat{u} \) lies on the unit circle and you have
\[
\hat{u} = < x, y >= < \cos \theta, \sin \theta > = (\cos \theta)i + (\sin \theta)j
\]

The angle \( \theta \) is the ________________ of the vector \( u \).

If \( u \) is not a unit vector, then:
\[
\|v\| = \|v\| (\cos \theta)i + \|v\| (\sin \theta)j
\]

Because \( v = ai + bj = \|v\| (\cos \theta)i + \|v\| (\sin \theta)j \) it follows that the direction angle \( \theta \) for \( v \) is determined from
\[
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\|v\| \sin \theta}{\|v\| \cos \theta} = \frac{b}{a}
\]

Ex7) Find the magnitude and direction angle of:

a) \( w = 6(\cos 210^\circ i + \sin 210^\circ j) \) 

b) \( v = -3i + 4j \)
1) Find the vector $\mathbf{v}$ with the given magnitude and the same direction as $\mathbf{u}$ and sketch the vector.

$\|\mathbf{v}\| = 6 \quad \mathbf{u} = (-3,3)$

2) Find the magnitude and direction angle of $\mathbf{v} = 5\mathbf{i} - 6\mathbf{j}$
Using the vectors below, find (a) \( \mathbf{u} + \mathbf{v} \), (b) \( \mathbf{u} - \mathbf{v} \), and (c) \( 3\mathbf{u} - 2\mathbf{v} \). Then sketch your resultant vector.

\( \mathbf{u} = (-2, 1) \quad \mathbf{v} = (3, 1) \)

(a) \( \mathbf{u} + \mathbf{v} = \langle 1, 2 \rangle \)

(b) \( \mathbf{u} - \mathbf{v} = \langle -5, 0 \rangle \)

(c) \( 3\mathbf{u} - 2\mathbf{v} = \langle -6, 3 \rangle - \langle 6, 2 \rangle \)

\( = \langle 0, 1 \rangle \)

\[ \text{Graphs showing vectors and their operations.} \]
Formulas learned so far

Component Form of a Vector: \( \vec{v} = \vec{PQ} = (q_1 - p_1, q_2 - p_2) = (v_x, v_y) \)

Magnitude of a vector: \( ||\vec{v}|| = \sqrt{v_x^2 + v_y^2} \)

Think about it

How many possible vectors can you find in one given direction? **Infinitely Many**

How many possible vectors can you find in one given direction that have a magnitude of 1? **One**

A **unit vector** is a vector that has the same direction of a vector \( \vec{v} \) and a magnitude of 1.

Note: A unit vector is usually indicated with a \( \hat{\text{v}} \) over the letter denoting the vector, for example: \( \hat{i} \)

It is often useful to find a unit vector in the same direction as a given vector \( \vec{v} \). To do this, divide \( \vec{v} \) by its magnitude.

\[
\hat{\text{v}} = \text{unit vector} = \frac{\vec{v}}{||\vec{v}||} = \left( \frac{1}{||\vec{v}||} \right) \vec{v}
\]

Ex1) Find a unit vector in the direction of \( \vec{v} = \langle -3, 5 \rangle \) and verify that the resulting vector has a magnitude of 1.

\[
\hat{\text{v}} = \left( \frac{-3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right)
\]

Ex2) Find a unit vector in the direction of \( \vec{v} = \langle 5, -12 \rangle \) and verify that the resulting vector has a magnitude of 1.

\[
\hat{\text{v}} = \left( \frac{5}{13}, \frac{-12}{13} \right)
\]
Ex3) Find the vector \( \vec{v} \) with the given magnitude and the same direction as \( \vec{u} \) and sketch the vector.

\[ ||\vec{v}|| = 5 \quad \vec{u} = (3,3) \]

\[ \vec{v} = \left< \frac{60}{8} \frac{78}{8} \right> \]

Ex4) Find the vector \( \vec{v} \) with the given magnitude and the same direction as \( \vec{u} \) and sketch the vector.

\[ ||\vec{v}|| = 9 \quad \vec{u} = (2,5) \]

\[ \vec{v} = \left< \frac{180}{8} \frac{150}{8} \right> \]

The unit vectors \( \langle 1, 0 \rangle \) and \( \langle 0, 1 \rangle \) are called the standard unit vectors and are denoted by \( \vec{i} = \langle 1, 0 \rangle \) and \( \vec{j} = \langle 0, 1 \rangle \).

Any vector in the plane can be written as a linear combination of the standard unit vectors.

Linear Combination: \( \vec{v} = a\vec{i} + b\vec{j} \)

Ex5) Write the vector \( \vec{v} = \langle -5, 3 \rangle \) as a linear combination.

\[ \vec{v} = -5\vec{i} + 3\vec{j} \]

Ex6) Let \( \vec{u} = -3\vec{i} + 8\vec{j} \) and \( \vec{v} = 2\vec{i} - \vec{j} \). Find \( 2\vec{u} - 3\vec{v} \).

\[ 2\vec{u} - 3\vec{v} = -12\vec{i} + 19\vec{j} \]
If \( \hat{u} \) is a unit vector such that \( \theta \) is the angle (measured clockwise) from the positive x-axis to \( \hat{u} \), the terminal point of \( \hat{u} \) lies on the unit circle and you have \( \hat{u} = \langle x, y \rangle = \langle \cos \theta, \sin \theta \rangle = (\cos \theta)\hat{i} + (\sin \theta)\hat{j} \)

The angle \( \theta \) is the direction angle of the vector \( \mathbf{u} \).

If \( \mathbf{u} \) is not a unit vector, then:

\[
\mathbf{v} = ||\mathbf{v}|| \langle \cos \theta, \sin \theta \rangle = ||\mathbf{v}|| \langle \cos \theta \rangle \hat{i} + ||\mathbf{v}|| \langle \sin \theta \rangle \hat{j}
\]

Because \( \mathbf{v} = a\hat{i} + b\hat{j} = ||\mathbf{v}|| \langle \cos \theta \rangle \hat{i} + ||\mathbf{v}|| \langle \sin \theta \rangle \hat{j} \) it follows that the direction angle \( \theta \) for \( \mathbf{v} \) is determined from

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \frac{||\mathbf{v}|| \sin \theta}{||\mathbf{v}|| \cos \theta} = \frac{b}{a}
\]

Ex7) Find the magnitude and direction angle of:

a) \( \mathbf{w} = 6(\cos 210^\circ \hat{i} + \sin 210^\circ \hat{j}) \)

\[
||\mathbf{w}|| = 6
\]

\( \theta = 210^\circ \)

b) \( \mathbf{v} = -3\hat{i} + 4\hat{j} \)

\[
||\mathbf{v}|| = 5
\]

\( \theta = 53.13^\circ \)
1) Find the vector \( \mathbf{v} \) with the given magnitude and the same direction as \( \mathbf{u} \) and sketch the vector.

\[
\|\mathbf{v}\| = 6 \quad \mathbf{u} = (-3, 3)
\]

\[
\mathbf{v} = \left< \frac{-6}{\sqrt{18}}, \frac{6}{\sqrt{18}} \right>
\]

2) Find the magnitude and direction angle of \( \mathbf{v} = 5\mathbf{i} - 6\mathbf{j} \)

\[
|\mathbf{v}| = \sqrt{61}
\]

\[
\theta = 50.10^\circ
\]
Homework Problems - Vectors Day 3

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*Find the vector \( \mathbf{v} \) with the given magnitude and the same direction as \( \mathbf{u} \).

38.) \( ||\mathbf{u}|| = 6 \)  \( \mathbf{u} = \langle -3, 3 \rangle \)  \( \text{Ans:} \) \( \mathbf{v} = \langle -\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}} \rangle \)

40.) \( ||\mathbf{u}|| = 10 \)  \( \mathbf{u} = \langle -10, 0 \rangle \)  \( \text{Ans:} \) \( \mathbf{v} = \langle -10, 0 \rangle \)

* Find \( 2\mathbf{u} - 3\mathbf{v} \) and sketch the resultant vector.

24.) \( \mathbf{u} = -2\mathbf{i} + \mathbf{j} \)  \( \mathbf{v} = -\mathbf{i} + 2\mathbf{j} \)  \( \text{Ans:} \) \( \mathbf{u} = -\mathbf{i} - 4\mathbf{j} \)

25.) \( \mathbf{u} = 2\mathbf{i} \)  \( \mathbf{v} = \mathbf{j} \)  \( \text{Ans:} \) \( \mathbf{u} = 4\mathbf{i} - 3\mathbf{j} \)

* Find the magnitude and direction angle of the vector \( \mathbf{v} \).

47.) \( \mathbf{v} = 3(\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}) \)  \( \text{Ans:} \) \( ||\mathbf{v}|| = 3 \quad \theta = 60^\circ \)

49.) \( \mathbf{v} = 6\mathbf{i} - 6\mathbf{j} \)  \( \text{Ans:} \) \( ||\mathbf{v}|| = 6\sqrt{2} \quad \theta = 315^\circ \)
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

### Component I: Classroom Teaching

#### Task A-2: Lesson Plan

Intern Name: Andrea Meadors, Stephen Powers, and Jessica Doering  

Date: 

Cycle: 

<table>
<thead>
<tr>
<th># of Students:</th>
<th>Age/Grade Level: 11th and 12th</th>
<th>Content Area: Pre-Calculus</th>
</tr>
</thead>
</table>

**Unit Title:** Vectors and their Applications  
**Lesson Title:** Direction Angles and Dot Products

**Lesson Alignment to Unit**

Respond to the following items:

a) Identify learning targets addressed by this lesson.

1. Students will find the component form of \( \vec{v} \) given its magnitude and the angle it makes with the positive x-axis.
2. Students will find the component form of the sum of \( \vec{u} \) and \( \vec{v} \) direction angles \( \theta_u \) and \( \theta_v \).
3. Students will compute the dot product of two vectors and explore the properties of the dot product.

b) Connect the learning targets to the state curriculum documents, i.e., Kentucky Core Academic Standards and the Mathematical Practices. List at least 2-3 target standards and at least 2 mathematical practices.

*Kentucky Core Academic Standards*

N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.

N.VM.5 (+) Multiply a vector by a scalar.

*Mathematical Practices*

M.P.1 Make sense of problems and persevere in solving them.
M.P.6 Attend to precision.
M.P.8 Look for and express regularity in repeated reasoning.

c) Describe students’ prior knowledge or focus of the previous learning.

Students have knowledge of vectors including how to find component form of a vector, how to find magnitude of a vector, how to graph vectors, how to perform vector operations, and how to find unit vectors and direction angles.

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The summative assessment comes in the form of a test over vectors and their applications. This lesson provides a mathematical representation, using direction angles and dot products, which then make it possible to talk about the applications of vectors.

e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit.

Throughout the lesson, the teacher will have opportunities to give students individual help that need it. Those that require extra time to complete assignments as stated in their IEPs will receive it. The teacher will handle students with additional diverse needs on an individual basis.
f) Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

The warm-up will provide a brief review of the previous day’s lesson. The lesson will build upon previously learned concepts such as how to find unit vectors and direction angles of vectors.

<table>
<thead>
<tr>
<th>Lesson Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective/Target:</strong> Find the component form of ( \vec{v} ) given its magnitude and the angle it makes with the positive x-axis.</td>
<td><strong>Assessment description:</strong> Addressed in previous lesson. Because of similarity to second objective, assessment is covered there. Exit Slip</td>
<td><strong>Strategy/Activity:</strong> Discuss previous learning about direction angles emphasizing right triangles</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong> See below</td>
<td><strong>Activity Adaptations:</strong> Guided notes available for lecture portion</td>
<td><strong>Media/technologies/resources:</strong> N/A</td>
</tr>
<tr>
<td><strong>Objective/Target:</strong> Find the component form of the sum of ( \vec{u} ) and ( \vec{v} ) direction angles ( \theta_u ) and ( \theta_v ).</td>
<td><strong>Assessment description:</strong> Student questioning and board presentation of example.</td>
<td><strong>Strategy/Activity:</strong> Leading questions and multiple visual representations are provide</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong> Students have time to work on the examples and can work with other students</td>
<td><strong>Activity Adaptations:</strong> Guided notes available for lecture portion</td>
<td><strong>Media/technologies/resources:</strong> N/A</td>
</tr>
<tr>
<td><strong>Objective/Target:</strong> Compute the dot product of two vectors and explore the properties of the dot product.</td>
<td><strong>Assessment description:</strong> Student questioning, shared examples, and Exit Slip</td>
<td><strong>Strategy/Activity:</strong> Class discussion</td>
</tr>
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<td><strong>Media/technologies/resources:</strong> N/A</td>
</tr>
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</table>

Procedures: Describe the sequence of strategies and activities you will use to engage students and accomplish your objectives. Within this sequence, describe how the differentiated strategies will meet individual student needs and diverse learners in your plan. (Use this section to outline the who, what, when, and where of the instructional strategies and activities.)

I. Warm up/ Bell ringer
   A) See attached bell ringer sheet
   B) Review Answers

II. Direction Angle
   A) Given magnitude and direction angle, find component form.

   \[
   u_1 = \|\vec{u}\| \sin \theta \\
   u_2 = \|\vec{u}\| \cos \theta 
   \]

   Graph over here

   B) Given magnitude and direction angle of \( \vec{u} \) and \( \vec{v} \) find the component form of \( \vec{u} + \vec{v} \).
1. Recall Moment
   If \( \vec{u} = \langle u_1, u_2 \rangle \) and \( \vec{v} = \langle v_1, v_2 \rangle \), then \( \vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle \)

2. Guided Example- Given \( \|\vec{u}\| = 20, \theta_u = 45^\circ, \|\vec{v}\| = 50, \) and \( \theta_v = 180^\circ \), find the component form of \( \vec{u} + \vec{v} \).
   \[ \vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2 \rangle \]
   \[ u_1 = 20 \cos 45^\circ \quad v_1 = 50 \cos 180^\circ = 50(-1) = -50 \]
   \[ u_2 = 20 \sin 45^\circ \quad v_2 = 50 \sin 180^\circ = 50(0) = 0 \]
   \[ \vec{u} + \vec{v} = (20 \cos 45^\circ - 50, 20 \sin 45) \]
   \[ \vec{u} + \vec{v} = (-35.86, 14.14) \]

C) General Form
   \[ \vec{u} + \vec{v} = (\|\vec{u}\| \cos \theta_u + \|\vec{v}\| \cos \theta_v, \|\vec{u}\| \sin \theta_u + \|\vec{v}\| \sin \theta_v) \]

III. Dot Product
   A) Definition-
   1. For \( \vec{u} = \langle u_1, u_2 \rangle \) and \( \vec{v} = \langle v_1, v_2 \rangle \), \( \vec{u} \cdot \vec{v} = u_1(v_1) + u_2(v_2) \)
   2. Examples
      a) For \( \vec{u} = \langle 3, -2 \rangle \) and \( \vec{v} = \langle 4, 5 \rangle \), find \( \vec{u} \cdot \vec{v} \)
         \( (3, -2) \cdot (4, 5) = 3(4) + (-2)(5) = 12 - 10 = 2 \)
      b) For \( \vec{u} = 2i + j \) and \( \vec{v} = 5i - 6j \), find \( \vec{u} \cdot \vec{v} \)
         \( \vec{u} \cdot \vec{v} = 2(5) + 1(-6) = 10 - 6 = 4 \)

   3. Note- Vector + Vector = Vector BUT.....
      Vector \cdot Vector = Scalar

B) Properties of Dot Product- For vectors \( \vec{u}, \vec{v}, \vec{w}, \text{ and } \vec{w}, \) and scalar \( a \)
   1) \( \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \) \hspace{1cm} \text{Commutative Property}
   2) \( (a\vec{u}) \cdot \vec{v} = a(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (a\vec{v}) \) \hspace{1cm} \text{Associative Property}
   3) \( (\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} \) \hspace{1cm} \text{Distributive Property}
   4) \( \vec{u} \cdot \vec{u} = \|\vec{u}\|^2 \)
      a) Why is this last one true?
      \( \vec{u} = \langle u_1, u_2 \rangle \)
      So, \( \vec{u} \cdot \vec{u} = u_1(u_1) + u_2(u_2) = (u_1)^2 + (u_2)^2 = (\sqrt{(u_1)^2 + (u_2)^2})^2 = \|\vec{u}\|^2 \)
      This relationship will be important to what we cover tomorrow.

IV. Exit Slip
   A) See Attached
1) Find the vector $\mathbf{v}$ with the given magnitude and the same direction as $\mathbf{u}$ and sketch the vector.

$$\|\mathbf{v}\| = 2 \quad \mathbf{u} = (-8, 15)$$

2) Find the magnitude and direction angle of $\mathbf{v} = -7\mathbf{i} + 4\mathbf{j}$
1) Find the vector \( \mathbf{v} \) with the given magnitude and the same direction as \( \mathbf{u} \) and sketch the vector.

\[ \| \mathbf{v} \| = 2 \quad \mathbf{u} = (-8, 15) \]

\[ \mathbf{v} = \left( -\frac{16}{17}, \frac{30}{17} \right) \]

2) Find the magnitude and direction angle of \( \mathbf{v} = -7\mathbf{i} + 4\mathbf{j} \)

\[ \| \mathbf{v} \| = \sqrt{65} \]
\[ \theta = 150.26^\circ \]
Guided Notes
Vectors Day 4

I. Direction Angle
   A) Given magnitude and direction angle, find component form.
      \[ u_1 = \text{_________} \]
      \[ u_2 = \text{_________} \]

   B) Given magnitude and direction angle of \( \vec{u} \) and \( \vec{v} \) find the component form of \( \vec{u} + \vec{v} \).
      1. Recall Moment
         If \( \vec{u} = (u_1, u_2) \) and \( \vec{v} = (v_1, v_2) \), then \( \vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2) \)

      2. Guided Example- Given \( ||\vec{u}|| = 20, \theta_u = 45^\circ, ||\vec{v}|| = 50, \text{and } \theta_v = 180^\circ \), find the component form of \( \vec{u} + \vec{v} \).
         \[ \vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2) \]
         \[ u_1 = \text{_________} \quad v_1 = \text{_________} \]
         \[ u_2 = \text{_________} \quad v_2 = \text{_________} \]
         \( \vec{u} + \vec{v} = (\text{_________}, \text{_________}) \)
         \( \vec{u} + \vec{v} = (\text{_________}, \text{_________}) \)

   C) General Form
      \( \vec{u} + \vec{v} = (\text{_________}, \text{_________}) \)

II. Dot Product
   A) Definition-
      1. For \( \vec{u} = (u_1, u_2) \) and \( \vec{v} = (v_1, v_2) \), \( \vec{u} \cdot \vec{v} = \text{_________} \)
      2. Examples
         c) For \( \vec{u} = (3, -2) \) and \( \vec{v} = (4, 5) \), find \( \vec{u} \cdot \vec{v} \)
         d) For \( \vec{u} = 2\mathbf{i} + \mathbf{j} \) and \( \vec{v} = 5\mathbf{i} - 6\mathbf{j} \), find \( \vec{u} \cdot \vec{v} \)
3. **Note** - Vector + Vector = Vector  **BUT.....**
   Vector ∙ Vector = Scalar

B) **Properties of Dot Product** - For vectors \( \vec{u}, \vec{v}, \text{and} \ \vec{w}, \text{and scalar} \ a \)

1) \( \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u} \)  
   ___________________ Property

2) \( (a\vec{u}) \cdot \vec{v} = a(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (a\vec{v}) \)  
   ___________________ Property

3) \( (\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w} \)  
   ___________________ Property

4) \( \vec{u} \cdot \vec{u} = ||\vec{u}||^2 \)
   a) Why is this last one true?
   \( \vec{u} = (u_1, u_2) \)
   So, \( \vec{u} \cdot \vec{u} = \)

This relationship will be important to what we cover tomorrow.
Exit Slip       Vectors Day 4                        Name________________________

\[
\|\vec{u}\| = 30 \quad \theta_{\vec{u}} = 60^\circ \\
\|\vec{v}\| = 10 \quad \theta_{\vec{v}} = 30^\circ 
\]

1. Find the component form of \(\vec{u}\) and the component form of \(\vec{v}\)

2. Find \(\vec{u} \cdot \vec{v}\)
Exit Slip       Vectors Day 4                        Name_______Key________________

\[ \|\vec{u}\| = 30 \quad \theta_{\vec{u}} = 60^\circ \]
\[ \|\vec{v}\| = 10 \quad \theta_{\vec{v}} = 30^\circ \]

1. Find the component form of \(\vec{u}\) and the component form of \(\vec{v}\)

\[ \vec{u} = (30 \cos 60^\circ, 30 \sin 60^\circ) = (30 \left(\frac{1}{2}\right), 30 \left(\frac{\sqrt{3}}{2}\right)) = (15, 15\sqrt{3}) \]
\[ \vec{v} = (10 \cos 30^\circ, 10 \sin 30^\circ) = (10 \left(\frac{\sqrt{3}}{2}\right), 10 \left(\frac{1}{2}\right)) = (5\sqrt{3}, 5) \]

2. Find \(\vec{u} \cdot \vec{v}\)

\[ \vec{u} \cdot \vec{v} = 15(5\sqrt{3}) + 5(15\sqrt{3}) = 75\sqrt{3} + 75\sqrt{3} = 150\sqrt{3} \]
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

### Component I: Classroom Teaching

#### Task A-2: Lesson Plan

<table>
<thead>
<tr>
<th>Intern Name: Andrea Meadors, Stephen Powers, and Jessica Doering</th>
<th>Date:</th>
<th>Cycle:</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Students:</td>
<td>Age/Grade Level: 11&lt;sup&gt;th&lt;/sup&gt; and 12&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Content Area: Pre-Calculus</td>
</tr>
<tr>
<td>Unit Title: Vectors and their Applications</td>
<td>Lesson Title: Vector Projections and Applications</td>
<td></td>
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</tbody>
</table>

**Lesson Alignment to Unit**

Respond to the following items:

a) Identify learning targets addressed by this lesson.

1. I can describe the angle formed between two vectors given the two vectors and write an equation for $\theta$ in terms of dot products and magnitudes.
2. I can find and explain what is meant by the component of $\mathbf{u}$ along $\mathbf{v}$.
3. I can find the projection of $\mathbf{u}$ onto $\mathbf{v}$ and describe how it relates to the component of $\mathbf{u}$ along $\mathbf{v}$.
4. I can make real world connections to vectors, and use the properties of vectors to describe the forces acting on an object at rest.

b) Connect the learning targets to the state curriculum documents, i.e., Kentucky Core Academic Standards and the Mathematical Practices. List at least 2-3 target standards and at least 2 mathematical practices.

**Kentucky Core Academic Standards**

- N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.
- N.VM.4 (+) Add and subtract vectors.
- N.VM.5 (+) Multiply a vector by a scalar.

**Mathematical Practices**

- M.P.1 Make sense of problems and persevere in solving them.
- M.P.2 Reason abstractly and quantitatively.
- M.P.4 Model with mathematics
- M.P.5 Use appropriate tools strategically.

c) Describe students’ prior knowledge or focus of the previous learning.

Students have knowledge of vectors including how to find component form of a vector, how to find magnitude of a vector, how to graph vectors, how to perform vector operations, how to find unit vectors and direction angles, and how to find the dot product of two vectors.

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The summative assessment comes in the form of a test over vectors and their applications. This lesson makes use of the content students have been learning about vectors to allow them to explore some real world applications of vectors.

e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit.
Throughout the lesson, the teacher will have opportunities to give students individual help that they need. Those that require extra time to complete assignments as stated in their IEPs will receive it. The teacher will handle students with additional diverse needs on an individual basis.

f)  Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

The warm-up will provide a brief review of the previous day’s lesson. The lesson will build upon previously learned concepts such as how to find component form of a vector given its magnitude and direction angle.

<table>
<thead>
<tr>
<th>Lesson Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
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<tbody>
<tr>
<td><strong>Objective/Target:</strong> I can describe the angle formed between two vectors given the two vectors and write an equation for ( \theta ) in terms of dot products and magnitudes</td>
<td><strong>Assessment description:</strong> Student questioning and exploration through visual representation. Exit Slip&lt;br&gt;<strong>Assessment Accommodations:</strong> Student collaboration, except on Exit Slip</td>
<td><strong>Strategy/Activity:</strong> Student and teacher led discussion, lecture and notes&lt;br&gt;<strong>Activity Adaptations:</strong> Guided notes available&lt;br&gt;<strong>Media/technologies/resources:</strong> N/A</td>
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<tr>
<td><strong>Objective/Target:</strong> I can find and explain what is meant by the component of ( \vec{u} ) along ( \vec{v} ).</td>
<td><strong>Assessment description:</strong> Student questioning and exploration through visual representation.&lt;br&gt;<strong>Assessment Accommodations:</strong> Student collaboration.</td>
<td><strong>Strategy/Activity:</strong> Student and teacher led discussion, lecture and notes&lt;br&gt;<strong>Activity Adaptations:</strong> Guided notes available&lt;br&gt;<strong>Media/technologies/resources:</strong> N/A</td>
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<td><strong>Objective/Target:</strong> I can find the projection of ( \vec{u} ) onto ( \vec{v} ) and describe how it relates to the component of ( \vec{u} ) along ( \vec{v} ).</td>
<td><strong>Assessment description:</strong> Student questioning and exploration through visual representation. Exit Slip&lt;br&gt;<strong>Assessment Accommodations:</strong> Student collaboration, except on Exit Slip</td>
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<td><strong>Objective/Target:</strong> I can make real world connections to vectors, and use the properties of vectors to describe the forces acting on an object at rest</td>
<td><strong>Assessment description:</strong> Two stations with guided worksheets that allow students to set up equations describing the interactions of the vectors. Questions at the end of the worksheet lets students create the equations and make extensions&lt;br&gt;<strong>Assessment Accommodations:</strong> Teacher is available for assistance</td>
<td><strong>Strategy/Activity:</strong> Interdisciplinary activity allowing students to see how vectors are used to describe the forces on an object&lt;br&gt;<strong>Activity Adaptations:</strong> Physical representations of each situations are presented, and teacher will be circulating to assist with set up&lt;br&gt;<strong>Media/technologies/resources:</strong> See activity for materials and set up</td>
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</table>
I. Warm up/Bell ringer
   A) See attached
   B) Go over answers to the bell ringer.

II. Angle between two vectors.
   A) Recall Moment.
      1) Dot Product.
         \[ \overrightarrow{u} \cdot \overrightarrow{v} = u_1(v_1) + u_2(v_2) \]
      2) The last property of dot products
         \[ \|\overrightarrow{u}\|^2 = \overrightarrow{u} \cdot \overrightarrow{u} \]

   B) Dot Product Theorem
      1) This provides a way to relate the magnitudes, dot product, and the angle the two vectors make with each other.
         \[ \overrightarrow{u} \cdot \overrightarrow{v} = \|\overrightarrow{u}\| \|\overrightarrow{v}\| \cos \theta \]

   C) Why???????
      1) Law of Cosines:
         \[ c^2 = a^2 + b^2 - 2ab \cos \theta \]
      2) In the triangle on the board, what represents \( c \)?
         \[ b? \|\overrightarrow{u} - \overrightarrow{v}\| \]
         \[ a? \|\overrightarrow{u}\| \]
      3) This creates \( \|\overrightarrow{u} - \overrightarrow{v}\|^2 = \|\overrightarrow{u}\|^2 + \|\overrightarrow{v}\|^2 - 2\|\overrightarrow{u}\|\|\overrightarrow{v}\| \cos \theta \)
      4) What have we learned that we can set \( \|\overrightarrow{u} - \overrightarrow{v}\|^2 \) equal to?
         a) \( \|\overrightarrow{u} - \overrightarrow{v}\|^2 = (\overrightarrow{u} - \overrightarrow{v}) \cdot (\overrightarrow{u} - \overrightarrow{v}) = \overrightarrow{u} \cdot \overrightarrow{u} - \overrightarrow{u} \cdot \overrightarrow{v} - \overrightarrow{v} \cdot \overrightarrow{u} + \overrightarrow{v} \cdot \overrightarrow{v} \)
         b) Distributive Property \( \rightarrow (\overrightarrow{u} - \overrightarrow{v}) \cdot (\overrightarrow{u} - \overrightarrow{v}) = \overrightarrow{u} \cdot \overrightarrow{u} - \overrightarrow{u} \cdot \overrightarrow{v} - \overrightarrow{v} \cdot \overrightarrow{u} + \overrightarrow{v} \cdot \overrightarrow{v} \)
         c) Last Property \( \rightarrow \|\overrightarrow{u}\|^2 - 2(\overrightarrow{u} \cdot \overrightarrow{v}) + \|\overrightarrow{v}\|^2 \)
      5) So, \( \|\overrightarrow{u}\|^2 - 2(\overrightarrow{u} \cdot \overrightarrow{v}) + \|\overrightarrow{v}\|^2 = \|\overrightarrow{u}\|^2 + \|\overrightarrow{v}\|^2 - 2\|\overrightarrow{u}\|\|\overrightarrow{v}\| \cos \theta \)
         \[ \overrightarrow{u} \cdot \overrightarrow{v} = \|\overrightarrow{u}\|\|\overrightarrow{v}\| \cos \theta \]

   D) Rewrite to get in terms of \( \theta \)
      \[ \cos \theta = \frac{\overrightarrow{u} \cdot \overrightarrow{v}}{\|\overrightarrow{u}\|\|\overrightarrow{v}\|} \]
      Now we can find the angle between two given vectors

   E) Quick check
      Find the angle between the vectors \( \overrightarrow{u} = \langle 2,5 \rangle \) and \( \overrightarrow{v} = \langle 4, -3 \rangle \)
      \[
      \cos \theta = \frac{2(4) + 5(-3)}{\sqrt{2^2 + 5^2} \sqrt{4^2 + (-3)^2}} \\
      = \frac{8 - 15}{\sqrt{29} \sqrt{25}} \\
      = \frac{-7}{5\sqrt{29}} \\
      \theta = \cos^{-1}\left(\frac{-7}{5\sqrt{29}}\right) \approx 105.1^\circ
      
      \]

If we have nonzero vectors, what would have to be true for the dot product to be 0?
1) $\cos \theta = 0$
   ONLY HAPPENS AT $\frac{\pi}{2}$ and $\frac{3\pi}{2}$
2) So, two nonzero vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u} \cdot \vec{v} = 0$
   Another name for perpendicular vectors is orthogonal

III. The component of $\vec{u}$ along $\vec{v}$
Also called the component of $\vec{u}$ in the direction of $\vec{v}$

A) Using right triangle trig...
1) Then component $\vec{u}$ along $\vec{v} = ||\vec{u}|| \cos \theta$
2) What kind of value does this produce
   a) Scalar

B) Where is this seen?
1) Car parked on a sloped drive way.
   a) Think of the weight of the car as a vector that always points straight down.
   b) Recall Moment
      i. Any vector can be written as the sum of two vectors
      c) In this case we want to write our vector, $\vec{w}$, as the sum of two perpendicular vectors.
         i. What direction might we want to point one of the vectors?
         ii. What will be the direction of the other vector then?
   d) What does knowing the magnitude of these vectors tell us?
      i. $\vec{u}$ tells us how much of the force or weight of the car is wanting to pull it down the hill
      ii. $\vec{v}$ tells us how much of the force or weight of the car is pushing directly into the ground

C) Using Dot Product to find the component of $\vec{u}$ along $\vec{v}$.
1) We know that $||\vec{u}|| ||\vec{v}|| \cos \theta = \vec{u} \cdot \vec{v}$
   and the component of $\vec{u}$ along $\vec{v} = ||\vec{u}|| \cos \theta$
   a) What can be done to the component equation to make it look like the dot product?
      i. Multiply by $\frac{||\vec{v}||}{||\vec{v}||} = 1$
         So $\frac{||\vec{u}|| ||\vec{v}|| \cos \theta}{||\vec{v}||} = \frac{||\vec{u}|| \cos \theta}{||\vec{v}||} = \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||}$

D) Example
1) Let $\vec{u} = (1,4)$ and $\vec{v} = (-2,1)$. Find the component of $\vec{u}$ along $\vec{v}$.
   $\vec{u} \cdot \vec{v} = 1(-2) + 4(1) = -2 + 4 = 2$
   $||\vec{v}|| = \sqrt{(-2)^2 + (1)^2} = \sqrt{4 + 1} = 2$

IV. The Projection of $\vec{u}$ onto $\vec{v}$
A) The component allows us to do two things
1) Break apart a vector into orthogonal vectors by selecting specific directions
2) Provides a scalar value

B) Projection of $\vec{u}$ onto $\vec{v}$ allows us to express the component of $\vec{u}$ along $\vec{v}$ in the direction of $\vec{v}$.
1) Because it produces a magnitude and direction, this creates a vector!!!!!!!!!!!!
2) Denoted $proj_{\vec{v}} \vec{u}$
C) Recall Moment
   1) How do we find a vector with a given magnitude in the direction of a vector \( \vec{v} \)?
      Multiply the unit vector of \( \vec{v} \) by the given magnitude.

D) So \( \text{proj}_v \vec{u} = (\text{component of } \vec{u} \text{ along } \vec{v})(\text{unit vector in direction of } \vec{v}) \)
   1) The component of \( \vec{u} \) along \( \vec{v} \) is \( \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2} \) and \( \vec{v} = \frac{\vec{v}}{||\vec{v}||} \).
      \[
      \text{proj}_v \vec{u} = \left( \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^2} \right) \left( \frac{\vec{v}}{||\vec{v}||} \right) = \left( \frac{\vec{u} \cdot \vec{v}}{||\vec{v}||^3} \right) \vec{v}
      \]
   2) This allows us to break/resolve \( \vec{u} \) into the sum of orthogonal vectors, \( \vec{u}_1 \) which is parallel to \( \vec{v} \), and \( \vec{u}_2 \) which is perpendicular to \( \vec{v} \).
      So if \( \vec{u}_1 = \text{proj}_v \vec{u} \), then \( \vec{u}_2 = \vec{u} - \text{proj}_v \vec{u} \)

E) Example
   1) Let \( \vec{u} = (-2, 9) \) and \( \vec{v} = (-1, 2) \)
      a) Find \( \text{proj}_v \vec{u} \)
      \[
      \text{proj}_v \vec{u} = \frac{-2(-1) + 9(2)}{(-1)^2 + (2)^2} \cdot (-1, 2) = \frac{2 + 18}{1 + 4} \cdot (-1, 2) = \frac{20}{5} \cdot (-1, 2) = 4(-1, 2) = (-4, 8)
      \]
      b) Resolve \( \vec{u} \) into \( \vec{u}_1 \) and \( \vec{u}_2 \) where \( \vec{u}_1 \) is parallel to \( \vec{v} \), and \( \vec{u}_2 \) is orthogonal to \( \vec{v} \).
         \( \vec{u}_1 = \text{proj}_v \vec{u} = (-4, 8) \) and \( \vec{u}_2 = \vec{u} - \text{proj}_v \vec{u} = (-2, 9) - (-4, 8) = (2, 1) \)

V. Forces at rest Activity
   A) See Attached Activity

VI. If Time
   A) Discuss the last question on the worksheet from Station 2

VII. Closure
   A) Hand out Exam Review (see attached)
1. Use the definition of dot product and component form, show that $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

2. $\vec{u} + \vec{v} = (4, 7), \|\vec{u}\| = 6, \|\vec{v}\| = 2, \theta_{\vec{u}} = 30^\circ$. Find $\theta_{\vec{v}}$. 
I. Angle between two vectors.
   A) Recall Moment.
      1) Dot Product.
         \[ \mathbf{u} \cdot \mathbf{v} = u_1(v_1) + u_2(v_2) \]
      2) The last property of dot products
         \[ \|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u} \]
   B) Dot Product Theorem
      1) This provides a way to relate the magnitudes, dot product, and the angle the two vectors make with each other.
      2) \( \mathbf{u} \cdot \mathbf{v} = \) ________________
   C) Why?
      1) Law of Cosines:
         \[ c^2 = a^2 + b^2 - 2ab \cos \theta \]
      2) In the triangle on the board, what represents \( c? \) ________________
         \( b? \) ________________
         \( a? \) ________________
      3) This creates _________________________________
      4) What have we learned that we can set \( \|\mathbf{u} - \mathbf{v}\|^2 \) equal to?
         d) \( \|\mathbf{u} - \mathbf{v}\|^2 = \) ________________
         e) Distributive Property \( \rightarrow \) __________________________
         f) Last Property \( \rightarrow \) __________________________
      5) So, \( \|\mathbf{u}\|^2 - 2(\mathbf{u} \cdot \mathbf{v}) + \|\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\|\mathbf{v}\| \cos \theta \)
         __________________________
   D) Rewrite to get in terms of \( \theta \)
      \[ \cos \theta = \] ________________
      Quick check
      Find the angle between the vectors \( \mathbf{u} = \langle 2, 5 \rangle \) and \( \mathbf{v} = \langle 4, -3 \rangle \)
      \( \cos \theta = \)
E) If we have nonzero vectors, what would have to be true for the dot product to be 0?
1) _______ = 0
   ONLY HAPPENS AT \( \frac{\pi}{2} \) and \( \frac{3\pi}{2} \)
2) So, two nonzero vectors \( \vec{u} \) and \( \vec{v} \) are perpendicular if and only if \( \vec{u} \cdot \vec{v} = 0 \)
   Another name for perpendicular vectors is ___________________

II. The component of \( \vec{u} \) along \( \vec{v} \)
   Also called the component of \( \vec{u} \) in the direction of \( \vec{v} \)

A) Using right triangle trig...
   1) Then component \( \vec{u} \) along \( \vec{v} \) = ________________

   2) What kind of value does this produce
      b) ________________

B) Where is this seen?
   1) Car parked on a sloped drive way.
      a) Think of the weight of the car as a vector that always points straight down.
      b) Recall Moment
         i. Any vector can be written as the sum of two vectors
      c) In this case we want to write our vector, \( \vec{w} \), as the sum of two perpendicular vectors.
         i. What direction might we want to point one of the vectors?
         ii. What will be the direction of the other vector then?
      d) What does knowing the magnitude of these vectors tell us?
         i. \( \vec{u} \) tells us how much of the force or weight of the car is wanting to pull it down the hill
         ii. \( \vec{v} \) tells us how much of the force or weight of the car is pushing directly into the ground

C) Using Dot Product to find the component of \( \vec{u} \) along \( \vec{v} \).
   1) We know that \( \|\vec{u}\|\|\vec{v}\| \cos \theta = \vec{u} \cdot \vec{v} \)
      and the component of \( \vec{u} \) along \( \vec{v} \) = \( \|\vec{u}\| \cos \theta \)
      a) What can be done to the component equation to make it look like the dot product?
         i. Multiply by _______ = 1

         So (____)\( \|\vec{u}\| \cos \theta = \) ________________ = ____________

D) Example
I. Let \( \vec{u} = (1,4) \text{ and } \vec{v} = (-2, 1) \). Find the component of \( \vec{u} \) along \( \vec{v} \).

III. The Projection of \( \vec{u} \) onto \( \vec{v} \)

A) The component allows us to do two things
1) Break apart a vector into orthogonal vectors by selecting specific directions
2) Provides a scalar value

B) Projection of \( \vec{u} \) onto \( \vec{v} \) allows us to express the component of \( \vec{u} \) along \( \vec{v} \) in the direction of \( \vec{v} \).
1) Because it produces a magnitude and direction, this creates a vector!
2) Denoted \( \text{proj}_v \vec{u} \)

C) Recall Moment
1) How do we find a vector with a given magnitude in the direction of a vector \( \vec{v} \)?

D) So \( \text{proj}_v \vec{u} = (\text{component of } \vec{u} \text{ along } \vec{v})(\text{unit vector in direction of } \vec{v}) \)
1) The component of \( \vec{u} \) along \( \vec{v} \) = \( \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} \) and \( \vec{v} = \frac{\vec{v}}{\|\vec{v}\|} \)

\[
\text{proj}_v \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|}
\]

2) This allows us to break/resolve \( \vec{u} \) into the sum of orthogonal vectors, \( \vec{u}_1 \) which is parallel to \( \vec{v} \), and \( \vec{u}_2 \) which is perpendicular to \( \vec{v} \).

So if \( \vec{u}_1 = \text{proj}_v \vec{u} \), then \( \vec{u}_2 = \vec{u} - \text{proj}_v \vec{u} \)

E) Example
1) Let \( \vec{u} = (-2, 9) \text{ and } \vec{v} = (-1, 2) \)
   a) Find \( \text{proj}_v \vec{u} \)

b) Resolve \( \vec{u} \) into \( \vec{u}_1 \) and \( \vec{u}_2 \) where \( \vec{u}_1 \) is parallel to \( \vec{v} \), and \( \vec{u}_2 \) is orthogonal to \( \vec{v} \).
Vector Applications Activity

Name: ________________________

Station 1)

Scenario: A boat is tied to a dock in the middle of the river. The current is trying to pull the boat straight down the river so that the rope is pulled tightly. We want to find out how much force the rope is exerting on the boat to keep it from floating down river.

Supplies: Tub of water, Newton scale, Toy boat, String

Set Up: The string should be attached to the top of the tub and the toy boat.

Procedure: Attach the scale to the back of the boat, and begin slowly pulling straight back until the string gets tight. Record the value of the scale as the force of the current acting on the boat and use the protractor to record the angle the string makes with the surface of the water.

Force = ___________________                      Angle = ___________________

Questions

1) Write the vector the represents the force experienced by the rope, \( \vec{F} \), as the sum of two vectors.

______________________________

2) Draw, and label, a picture that represents what is happening in this scenario.

3) How is the force of the current on the boat related to the force of the rope on the boat? (Hint: Think about why the boat is not floating down the river)

4) Write an equation to find the force acting on the rope. (Hint: Think about the component of \( \vec{F} \) onto \( \vec{v} \))

5) What is the force of the rope on the boat? \( ||\vec{F}|| = \) __________

Station 2)

Scenario: Two cranes are lifting and exceptionally heavy object. The cranes are set up on opposite sides of the object, and when the object is held stationary, the force that the crane on the right can lift straight up must not be more than one half that of the crane on the left to safely operate. We want to explore how safe it is for the two cranes to lift the object together.

Supplies: Two pulleys, String, Newton scale, Weight, Protractor
Set Up: Suspend the pulleys on opposite sides of an open space and attach the weight in between the pulleys. Make sure to leave enough slack in the string so that the angle the string makes with the ground can be adjusted.

Procedure: Use the Newton scale to find the force the weight will be exerting on the two cranes. Attach the weight to the two pulleys and adjust the angles the strings are making with the ground at the weight so that the angle is largest on the side of the left pulley. Record the force of the weight, and the angles that you chose. Readjust the pulleys so that this time the bigger angle is on the right side, again record.

<table>
<thead>
<tr>
<th>Trial 1</th>
<th>Trial 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W =$</td>
<td>$W =$</td>
</tr>
<tr>
<td>$\theta_1 =$</td>
<td>$\theta_1 =$</td>
</tr>
<tr>
<td>$\theta_2 =$</td>
<td>$\theta_2 =$</td>
</tr>
</tbody>
</table>

Questions

1) Which of these trials do you think will put the greatest vertical strain on the right pulley? Why?

2) Because the object is at rest, what can we say about the horizontal component of the tension in the pulleys? What about the vertical components? Write an equation to show these relationships. *(Hint: Think about the direction the horizontal components are in relation to each other)*

3) Using your equations, find the tension in both the right and left strings for each of your two trials

4) Using the tensions found in problem #3, find the vertical force each pulley is experiencing. Was your prediction in problem #1 correct?
5) Based on your findings in problem 4, around what angle do you think the crane on the right needs to be at for the object to be lifted? Choose a couple of angles in that range to see if you can find the breaking point.
Advanced Pre-Calculus – Vectors Review

Name: ____________________________

Note: The answers to the review problems are located at the end of the review to help you study. Return the completed review (with work shown!) tomorrow to receive 2 bonus points on the exam.

1. Perform the indicated vector addition graphically. Be sure to label all vectors, including the resultant.

Start at the origin.

2. A vector has initial point (2, –3) and terminal point (3, –7). Find its component form and magnitude.

3. Find a unit vector in the direction of \( w = 6i - 2j \).

4. A vector \( v \) has magnitude 430 and direction \( \theta = 150^\circ \), and vector \( w \) has a magnitude 42 and direction \( \theta = 240^\circ \). Find \( v + w \) and write the answer in “linear combination” form. \( (i, j) \) form

5. An airplane’s airspeed is 724 mph at a bearing of N 30° E. If the wind velocity is 32 mph from the west, find the groundspeed and the direction of the plane.
6. Given \( \vec{v} = \langle 2, 5 \rangle \) and \( \vec{w} = \langle -2, -3 \rangle \), find the angle \( \theta \) between \( \vec{v} \) and \( \vec{w} \).

7. Given that \( \vec{v} = 2\hat{i} + 6\hat{j} \) and \( \vec{w} = -8\hat{i} - 9\hat{j} \), find  
   a) \( 2\vec{v} + 3\vec{w} \)  
   b) \( \vec{v} \cdot \vec{w} \)  
   c) \( \vec{v} \cdot 3\vec{w} \)

8. Determine if the vectors \( \vec{v} = 2\hat{i} + 4\hat{j} \) and \( \vec{w} = 4\hat{i} - 2\hat{j} \) are orthogonal, parallel, or neither.

9. Given \( \vec{u} = \langle 2, -3 \rangle \) and \( \vec{v} = \langle 3, 6 \rangle \), find the projection of \( \vec{u} \) onto \( \vec{v} \).
Answers

1. \((-4, 6)\) and \((-6, -5)\)

2. \(\langle 1, -4 \rangle\) with magnitude of \(\sqrt{17}\)

3. \(\frac{3\sqrt{10}}{10}, -\frac{\sqrt{10}}{10}\)

4. \(-393.39i + 178.63j\)

5. 740.52 mph at a heading of \(N32.14^\circ E\)

6. 168.11°

7. a) \(-20i - 15j\) b) \(-70\) c) \(-210\)

8. Orthogonal b/c \(8 + (-8) = 0\)

9. \(\left\langle -\frac{4}{5}, -\frac{8}{5}\right\rangle\)
1. Use the definition of dot product and component form, show that $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

\[
\vec{u} \cdot \vec{v} = u_1(v_1) + u_2(v_2) = v_1(u_1) + v_2(u_2) = \vec{v} \cdot \vec{u}
\]

2. $\vec{u} + \vec{v} = (4,5)$, $\|\vec{u}\| = 6$, $\|\vec{v}\| = 3, \theta_{\vec{u}} = 30^\circ$. Find $\theta_{\vec{v}}$.

\[
\begin{align*}
5 &= 6 \sin 30^\circ + 3 \sin \theta_{\vec{v}} \\
5 &= 3 + 3 \sin \theta_{\vec{v}} \\
2 &= 3 \sin \theta_{\vec{v}} \\
\sin \theta_{\vec{v}} &= \frac{2}{3} \\
\theta_{\vec{v}} &= \sin^{-1} \frac{2}{3} = 41.8^\circ
\end{align*}
\]
Your committee members will review and evaluate your performance on this task using Standard 1: The teacher demonstrates applied content knowledge and Standard 2: The teacher designs and plans instruction.

**Component I: Classroom Teaching**

**Task A-2: Lesson Plan**

<table>
<thead>
<tr>
<th>Intern Name:</th>
<th>Andrea Meadors, Stephen Powers, and Jessica Doering</th>
<th>Date:</th>
<th>Cycle:</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Students:</td>
<td>Age/Grade Level: 11th and 12th</td>
<td>Content Area:</td>
<td>Pre-Calculus</td>
</tr>
<tr>
<td>Unit Title: Vectors and their Applications</td>
<td>Lesson Title: Vectors Exam</td>
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</tbody>
</table>

**Lesson Alignment to Unit**

Respond to the following items:

a) Identify learning targets addressed by this lesson.

1. Students will find the component form and magnitude of the vector $v$.
2. Students will perform vector addition and scalar multiplication.
3. Students will find unit vectors and direction angles of given vectors.
4. Students will find the component form of $v$ given its magnitude and the angle it makes with the positive x-axis.
5. Students will find the dot product of two vectors and the angle between two vectors.
6. Students will determine whether two vectors are orthogonal, parallel, or neither.
7. Students will find the projection of $u$ onto $v$.
8. Students will apply vectors to the real world.

b) Connect the learning targets to the state curriculum documents, i.e., Kentucky Core Academic Standards and the Mathematical Practices. List at least 2-3 target standards and at least 2 mathematical practices.

*Kentucky Core Academic Standards*

N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.

N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.4 (+) Add and subtract vectors.

N.VM.5 (+) Multiply a vector by a scalar.

*Mathematical Practices*

M.P.1 Make sense of problems and persevere in solving them.

M.P.2 Reason abstractly and quantitatively.

M.P.6 Attend to precision.

M.P.8 Look for and express regularity in repeated reasoning.

c) Describe students’ prior knowledge or focus of the previous learning.

Throughout the unit, students have learned all needed aspects of vectors and their applications. They will be able to demonstrate that knowledge for this assessment.

d) Describe summative assessment(s) for this particular unit and how lessons in this unit contribute to the summative assessment.

The lessons in this unit have prepared them for all content being assessed in this summative assessment, such as
component form of a vector, magnitude of a vector, unit vectors, direction angles, and dot products of vectors.

e) Describe the characteristics of your students identified in Task A-1 who will require differentiated instruction to meet their diverse needs impacting instructional planning in this lesson of the unit. Some students may require extra time to complete their exam or need to be placed in a secluded location so as not to be distracted by their peers. These accommodations will be made. All other needs will be determined on an individual basis.

f) Pre-Assessment: Describe your analysis of pre-assessment data used in developing lesson objectives/learning targets (Describe how you will trigger prior knowledge):

Students have previously been given formative assessments on all parts of the content learned in this unit. Their performance on these assessments has been used to help create this summative assessment.

<table>
<thead>
<tr>
<th>Lesson Learning Targets</th>
<th>Assessment</th>
<th>Instructional Strategy/Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective/Target:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Students will find the component form and magnitude of the vector $v$.</td>
<td>Assessment description: Summative Exam</td>
</tr>
<tr>
<td></td>
<td>Assessment Accommodations: Students who require extra time or a special location to complete their exam will receive it.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Media/technologies/resources: Handout, calculators</td>
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<td>Objective/Target:</td>
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<td>Strategy/Activity: Unit Exam</td>
</tr>
<tr>
<td></td>
<td>Assessment Accommodations: Students who require extra time or a special location to complete their exam will receive it.</td>
<td>Activity Adaptations: Students who need extra time will be given opportunities to finish at a later date.</td>
</tr>
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<tr>
<td>5. Students will find the dot product of two vectors and the angle between two vectors.</td>
<td>Summative Exam</td>
<td>Unit Exam</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong></td>
<td></td>
<td></td>
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<tr>
<td>6. Students will determine whether two vectors are orthogonal, parallel, or neither.</td>
<td>Summative Exam</td>
<td>Unit Exam</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Students will find the projection of $u$ onto $v$.</td>
<td>Summative Exam</td>
<td>Unit Exam</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong></td>
<td></td>
<td></td>
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<tr>
<td>8. Students will apply vectors to the real world.</td>
<td>Summative Exam</td>
<td>Unit Exam</td>
</tr>
<tr>
<td><strong>Assessment Accommodations:</strong></td>
<td></td>
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</tbody>
</table>

**Procedures:** Describe the sequence of strategies and activities you will use to engage students and accomplish your objectives. Within this sequence, describe how the differentiated strategies will meet individual student needs and diverse learners in your plan. (Use this section to outline the who, what, when, and where of the instructional strategies and activities.)

**Opener/Warm-Up (5 minutes)**

Students will have five minutes to look over their notes and review sheets.

**Lesson Development (45 minutes)**

Ask students to take everything off their desks except for a pencil and a calculator. Remind students that there is no talking during the exam. Hand out exams. Circulate around the room to ensure no students are cheating and to quietly answers any questions that students may have. Students will place exams into the appropriate tray for their class when finished.

**Closure (5 minutes)**
Remind students that there are only five minutes remaining and that they may have extra time to finish before school begins tomorrow if needed. Remind them to turn in their review worksheets for bonus points.

**If-Time Strategy:**
Ask students to quietly read their own books while others finish the exam or they may read ahead into the next chapter of the textbook to start thinking about the upcoming unit. They may not talk until all exams are turned in.
1. Given the vectors below, sketch the resultant vector. Be sure to label all parts.

2. A vector \( \mathbf{v} \) has initial point \((6, -3)\) and terminal point \((5, 9)\). Find the component form of vector \( \mathbf{v} \).

3. Determine the magnitude of \( \mathbf{v} \):
\[
\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}
\]

4. Find the direction of \( \mathbf{w} \):
\[
\mathbf{w} = -3\mathbf{i} - 5\mathbf{j}
\]
5. Given \( \mathbf{u} = -5\mathbf{i} + 2\mathbf{j}, \mathbf{v} = -6\mathbf{i} + 4\mathbf{j}, \) find \(-2\mathbf{u} + 2\mathbf{v}\)

\[ \text{[A]} \ 10\mathbf{i} + 8\mathbf{j} \quad \text{[B]} \ -13\mathbf{i} + 8\mathbf{j} \quad \text{[C]} \ -2\mathbf{i} + 4\mathbf{j} \quad \text{[D]} \ 22\mathbf{i} + 12\mathbf{j} \]

6. A vector \( \mathbf{v} \) has a magnitude 8 and direction \( \theta = 120^\circ \). Find \( \mathbf{v} \).

\[ \text{[A]} \ \left(-4, 4\sqrt{3}\right) \quad \text{[B]} \ \left(8\sqrt{3}, \frac{8\sqrt{3}}{3}\right) \quad \text{[C]} \ \left(4\sqrt{3}, -4\right) \quad \text{[D]} \ \left(\frac{8\sqrt{3}}{3}, -8\sqrt{3}\right) \]

7. Find a unit vector in the direction of \( \mathbf{v} = -5\mathbf{i} + 2\mathbf{j} \)

\[ \text{[A]} \ \left(-\frac{5\sqrt{21}}{21}, \frac{2\sqrt{21}}{21}\right) \quad \text{[B]} \ \left(-\frac{5\sqrt{29}}{29}, \frac{2\sqrt{29}}{29}\right) \quad \text{[C]} \ \left(-\frac{5\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right) \quad \text{[D]} \ \left(-1, 1\right) \]

8. An airplane is flying in the direction N \( 25^\circ \) W, with an airspeed of 500 mph. The wind is blowing from the north with a speed of 50 mph. What is the true direction of the plane and what is the groundspeed of the plane?

True direction ________________

Groundspeed ________________
9. Find the indicated dot product \( u \cdot v \) where \( u = -7i + 8j \) and \( v = 6i + 2j \)


10. Given \( v = \langle 5, 1 \rangle \) and \( w = \langle -4, 2 \rangle \), find the angle \( \theta \) between \( v \) and \( w \).

11. Which of the following pairs of vectors are orthogonal?

[A] \( v = 4i - 2j, w = 2i - 4j \)  
[B] \( v = 3i + j, w = 9i + 3j \)

[C] \( v = i - j, w = i \)  
[D] \( v = -2i + j, w = i + 2j \)

12. Given \( u = \langle 3, 2 \rangle \) and \( v = \langle 1, -3 \rangle \), find the projection of \( u \) onto \( v \).
1. Given the vectors below, sketch the resultant vector. Be sure to label all parts.

2. A vector $v$ has initial point $(6, -3)$ and terminal point $(-5, 9)$. Find the component form of vector $v$.

   [A] $\langle 1, -6 \rangle$  
   [B] $\langle 11, -12 \rangle$  
   [C] $\langle -1, 6 \rangle$  
   [D] $\langle -11, 12 \rangle$

3. Determine the magnitude of $v$: $v = -2i + 3j$

   [A] 13  
   [B] $\sqrt{13}$  
   [C] 5  
   [D] $\sqrt{5}$

4. Find the direction of $w$: $w = -3i - 5j$

   [A] 59°  
   [B] 121°  
   [C] 239°  
   [D] 301°
5. Given \( \mathbf{u} = -5 \mathbf{i} + 2 \mathbf{j}, \mathbf{v} = -6 \mathbf{i} + 4 \mathbf{j} \), find \(-2 \mathbf{u} + 2 \mathbf{v}\)

[A] \(10 \mathbf{i} + 8 \mathbf{j}\)  
[B] \(-13 \mathbf{i} + 8 \mathbf{j}\)  
[C] \(-2 \mathbf{i} + 4 \mathbf{j}\)  
[D] \(22 \mathbf{i} + 12 \mathbf{j}\)

6. A vector \( \mathbf{v} \) has a magnitude 8 and direction \( \theta = 120^\circ \). Find \( \mathbf{v} \).

[A] \((-4, \sqrt{3})\)  
[B] \((8\sqrt{3}, \frac{8\sqrt{3}}{3})\)  
[C] \((4\sqrt{3}, -4)\)  
[D] \(\left(\frac{8\sqrt{3}}{3}, -8\sqrt{3}\right)\)

7. Find a unit vector in the direction of \( \mathbf{v} = -5 \mathbf{i} + 2 \mathbf{j} \)

[A] \((-\frac{5\sqrt{21}}{21}, \frac{2\sqrt{21}}{21})\)  
[B] \((-\frac{5\sqrt{29}}{29}, \frac{2\sqrt{29}}{29})\)  
[C] \((-\frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5})\)  
[D] \((-1,1)\)

8. An airplane is flying in the direction N 25°W, with an airspeed of 500 mph. The wind is blowing from the north with a speed of 50 mph. What is the true direction of the plane and what is the groundspeed of the plane?

True direction __<\(-453.15, 161.31\)>_____

Groundspeed __481 mph______
9. Find the indicated dot product $u \cdot v$ where $u = -7i + 8j$ and $v = 6i + 2j$


10. Given $v = \langle 5, 1 \rangle$ and $w = \langle -4, 2 \rangle$, find the angle $\theta$ between $v$ and $w$. ___142.12°________

11. Which of the following pairs of vectors are orthogonal?

[A] $v = 4i - 2j, w = 2i - 4j$  [B] $v = 3i + j, w = 9i + 3j$
[C] $v = i - j, w = i$  [D] $v = -2i + j, w = i + 2j$

12. Given $u = \langle 3, 2 \rangle$ and $v = \langle 1, -3 \rangle$, find the projection of $u$ onto $v$. 

$\left( -\frac{3}{\sqrt{10}}, \frac{9}{\sqrt{10}} \right)$
References


